NMAG442 Representation Theory of Finite-Dimensional Algebras

Excercise session 7—May 19, 2022

Our goal today is to have a look at some examples of finite-dimensional algebras of infinite representation type, both tame and wild, and their representations.

We work over an algebraically closed k and with finite-dimensional modules.

Tame algebras

Exercise 1. Let us have the following representations $X_n, Y_n, T_n, n \geq 1$ of the Kronecker quiver:

$$X_n: \quad k^n \underset{\psi_n}{\overset{\varphi_n}{\rightrightarrows}} k^{n+1}$$

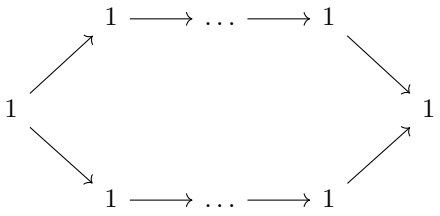
$$Y_n: \quad k^{n+1} \underset{\psi_n^T}{\overset{\varphi_n^T}{\rightrightarrows}} k^n$$

$$T_n: \quad k^n \underset{J_{n,\lambda}}{\overset{I_n}{\rightrightarrows}} k^n$$

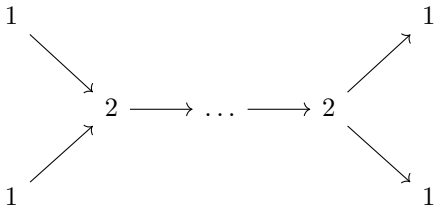
where φ_n is the inclusion on the first n coordinates, ψ_n is the inclusion on the last two coordinates, and $J_{\lambda,n}$ is the Jordan block pertaining to the scalar $\lambda \in k$ of size $n \times n$. Observe what effect repeated application of the reflection functors S^+ have.

Exercise 2 (cf. Section 4.2 in [2]). Find 1-dimensional families of indecomposable representations of the following Euclidean diagrams with given orientation with indicated dimension vectors:

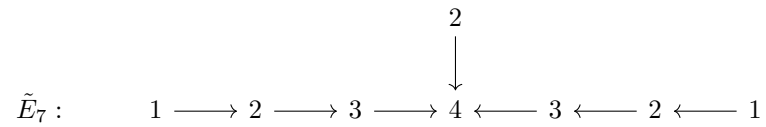
$\tilde{A}_m$, for $m \geq 0$:



$\tilde{D}_m$, for $m \geq 4$:



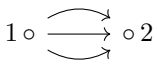
$\tilde{E}_7$:



(Hint: Use Proposition 2.5 in Chapter VII in [1].)

Wild algebras

Exercise 3 (Inspired by section 10.2 in [2]). Exhibit $1 - \langle \alpha, \alpha \rangle$-dimensional families of bricks (representations with only trivial endomorphisms) for dimension vectors α of $(1,1), (1,3)$ and $\{(n, n+1)\}_{n \geq 1}$ of the quiver $K(3)$:



(Hint: All the families may be parametrized by affine spaces of respective dimensions.)

References

[1] Assem, I., Skowronski, A., and Simson, D. *Elements of the Representation Theory of Associative Algebras: Volume 1: Techniques of Representation Theory*, vol. 65. Cambridge University Press, 2006.

[2] Krause, H. Representations of quivers via reflection functors. *arXiv preprint arXiv:0804.1428* (2008).

Feel free to reach me at `jakub.kopriva@mff.cuni.cz`. Also, I am available for short consultations on problems from the exercise sessions after previous arrangment via e-mail.