Representation Theory of Finite-Dimensional Algebras
NMAG442

Exercise session 1—February 23, 2023

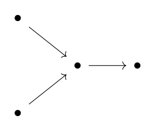
We work over an algebraically closed field k and with finite-dimensional modules.

Quiver representations and direct sum decomposition.

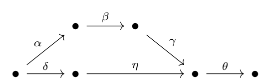
Exercise 1. For each of the following quivers, write the associated path algebra as a matrix algebra.

$$Q^{'} : \quad \bullet \longleftarrow \bullet \longrightarrow \bullet$$

$Q^{''}$:



$Q^{'''}$:



Let I be the ideal of $kQ^{'''}$ generated by the relation $\alpha\beta\gamma - \delta\eta$. Write the matrix algebra of $kQ^{'''}/I$.

Exercise 2. Let Q be a finite quiver. Prove that kQ is a finite dimensional $k-$algebra if and only if Q is finite and acyclic.

Exercise 3 (Subspace quiver). Let Q be a quiver with vertices labelled $0, \cdots, n$ and n arrows such that there is one arrow $i \to 0$ for each $1 \leq i \leq n$. Then, find:

a) An embedding $kQ \to M_{n+1}(k)$;

b) Natural direct decomposition for every module over kQ.

Exercise 4 (Kronecker quiver). For any $n \in \mathbb{N}$ find φ, ψ maps such that

$$k^n \underset{\psi}{\overset{\varphi}{\rightrightarrows}} k^{n+1}$$

is indecomposable. Compute the endomorphism ring of such a representation.

Exercise 5. Consider the following representaitons of the subspace quiver with 3 vertices:

$$M : \qquad k \xrightarrow{\begin{bmatrix}1\\0\end{bmatrix}} k^2 \xleftarrow{\begin{bmatrix}0\\1\end{bmatrix}} k$$

$$N : \qquad k \xrightarrow{\begin{bmatrix}1\\0\end{bmatrix}} k^2 \xleftarrow{\begin{bmatrix}1\\0\end{bmatrix}} k \ .$$

Show that these representations are neither indecomposable nor isomorphic.

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