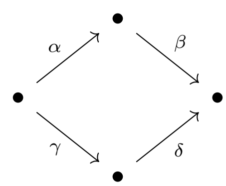
Representation Theory of Finite-Dimensional Algebras
NMAG442

Exercise session 2—March 9, 2023

We work over an algebraically closed field k and with finite-dimensional modules.

Admissible ideals, endomorphism ring and indecomposable representations.

Exercise 1. Let Q be the quiver



and $I_1 = \langle \alpha\beta + \gamma\delta \rangle$, $I_2 = \langle \alpha\beta - \gamma\delta \rangle$ two-sided ideals of kQ.

a) Decide whether I_1 and I_2 are admissible.

b) Show that there is an isomorphism $kQ/I_1 \cong kQ/I_2$.

Exercise 2. Let Q be the Kronecker quiver and M the following representation of Q

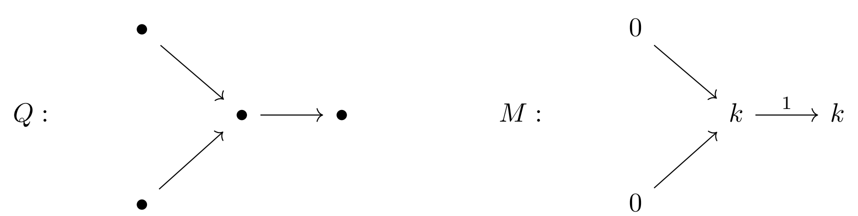
$$k^3 \underset{J_{3,0}}{\overset{1}{\rightrightarrows}} k^3$$

where 1 denotes the identity and $J_{3,0} = \begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$.

a) Compute $\mathrm{End}(M)$, the endomorphism ring of M.

b) Show that $\mathrm{End}(M)$ is a local ring.

Exercise 3. Consider the following quiver Q and his representation M



Compute $\mathrm{End}(M)$.

Exercise 4. Let A be a finite-dimensional algebra over k. Then, for every S simple module over A, show that $\mathrm{End}_A(S) \cong k$.

Exercise 5. a) Let A be a k-algebra. Show that an A-module M is indecomposable if and only if $\mathrm{End}_A(M)$ has no non-trivial idempotents.

b) Consider the quiver Q

$$\bullet \longrightarrow \bullet \longrightarrow \bullet$$

of type A_3. Find all the indecomposable representations of A_3.
(*Hint*: In total they are 6).

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