8. $\lim\limits_{x\to 0} \dfrac{\sin(a+2x) - 2\sin(a+x) + \sin a}{x^2} = \lim\limits_{x\to 0} \dfrac{[\sin(a+2x)-\sin(a+x)] - [\sin(a+x) - \sin a]}{x^2} =$

součet vzorec

$= \lim\limits_{x\to 0} \dfrac{1}{x^2}\left[2\sin\frac{x}{2}\cos\left(a+\frac{3x}{2}\right) - 2\sin\frac{x}{2}\cos\left(a+\frac{x}{2}\right)\right] =$

VOAL

$= \lim\limits_{x\to 0} \dfrac{\sin\frac{x}{2}}{\frac{x}{2}} \cdot \lim\limits_{x\to 0} \dfrac{\cos\left(a+\frac{3x}{2}\right) - \cos\left(a+\frac{x}{2}\right)}{x} = \left\Vert \begin{array}{l} \lim\limits_{x\to 0} \frac{x}{2} = 0 \ \& \ \frac{x}{2} \neq 0 \text{ na } P_\delta(0) \\ \lim\limits_{y\to 0} \frac{\sin y}{y} = 1 \quad \text{tj. z VOLSF je 1. limita rovna 1} \end{array} \right\Vert$

$= \lim\limits_{x\to 0} \dfrac{1}{x}\left[2\sin\frac{x}{2}\, \cancel{\cos}\sin(a+x)\right] \overset{\text{VOAL}}{=} \lim\limits_{x\to 0} \dfrac{\sin\frac{x}{2}}{\frac{x}{2}} \cdot \lim\limits_{x\to 0} \sin(a+x) = \underline{\underline{\sin a}}$

9. $\lim\limits_{x\to 0} \dfrac{\operatorname{cotg}(a+2x) - 2\operatorname{cotg}(a+x) + \operatorname{cotg} a}{x^2} = \lim\limits_{x\to 0} \dfrac{1}{x^2}\left\{[\operatorname{cotg}(a+2x) - \operatorname{cotg}(a+x)] - [\operatorname{cotg}(a+x) - \operatorname{cotg} a]\right\} =$

$\left\Vert \begin{array}{l} \operatorname{cotg}\alpha - \operatorname{cotg}\beta = \dfrac{\cos\alpha}{\sin\alpha} - \dfrac{\cos\beta}{\sin\beta} = \\ = \dfrac{\cos\alpha\sin\beta - \cos\beta\sin\alpha}{\sin\alpha\sin\beta} = \dfrac{-\sin(\alpha-\beta)}{\sin\alpha\sin\beta} \end{array} \right\Vert = \lim\limits_{x\to 0} \dfrac{1}{x^2}\left[\dfrac{-\sin x}{\sin(a+2x)\sin(a+x)} - \dfrac{-\sin x}{\sin(a+x)\sin a}\right] =$

VOAL

$= \lim\limits_{x\to 0} \dfrac{\sin x}{x} \lim\limits_{x\to 0} \dfrac{\sin(a+2x) - \sin a}{x\sin(a+2x)\sin(a+x)\sin a} \overset{\text{VOAL}}{=} \lim\limits_{x\to 0} \dfrac{1}{\sin(a+2x)\sin(a+x)\sin a} \lim\limits_{x\to 0} \dfrac{2\sin x\cos(a+x)}{x}$

základní limita

$\overset{\text{VOAL}}{=} \dfrac{1}{\sin^3 a} \cdot 2\cos a = \underline{\underline{\dfrac{2\cos a}{\sin^3 a}}}$

10. $\lim\limits_{x\to 0+} \dfrac{\arccos(1-x)}{\sqrt{x}} = \left\Vert \begin{array}{ll} \cos y = 1-x & x\to 0+ \\ x = 1-\cos y & 1-x \to 1- \\ & \arccos(1-x) = y \to 0+ \end{array} \right\Vert =$

$= \lim\limits_{y\to 0+} \dfrac{y}{\sqrt{1-\cos y}} \underset{\text{spojitost } \sqrt{\ }}{=} \sqrt{\lim\limits_{y\to 0+} \dfrac{y^2}{1-\cos y}} \overset{\text{VOAL}}{=} \sqrt{\dfrac{1}{\lim\limits_{y\to 0+} \frac{1-\cos y}{y^2}}} = \underline{\underline{\sqrt{2}}}$

$= \frac{1}{2}$ zákl. lim.