1. $\sqrt{x^2+2x-3} \geq \sqrt{x^2+3x-4}$ (1)

pre zaké $x \in \mathbb{R}$ platí ?

$x_1 x_2 = c$

$x_1 + x_2 = -b$

(a) $x^2+2x-3 \geq 0$

$= (x+3)(x-1)$

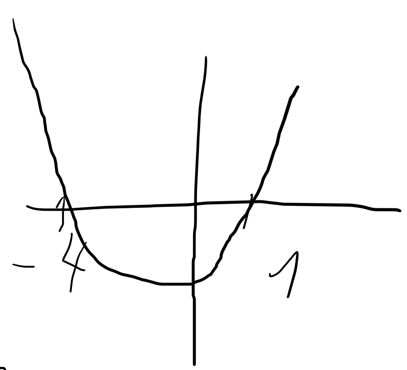
rovnica $x^2+2x-3=0$

$D = 4+12 = 16$

$x_{1,2} = \frac{-2 \pm \sqrt{16}}{2} = \frac{-2 \pm 4}{2} = \begin{cases} -3 \\ 1 \end{cases}$

$x \in (-\infty, -3] \cup [1, \infty)$

(b) $x^2+3x-4 \geq 0$, $x_1 = -4$, $x_2 = 1$



$x \in (-\infty, -4] \cup [1, \infty)$

Dohromady podmienky: $x \in (-\infty, -4] \cup [1, \infty)$

$(1) \Leftrightarrow \cancel{x^2}+2x-3 \geq \cancel{x^2}+3x-4$

$1 \geq x \Leftrightarrow x \in (-\infty, 1]$

riešenie: $x \in (-\infty, -4] \cup \{1\}$.

Znegujte a rozhodněte o pravdivosti:

1. $(\forall x, y \in \mathbb{R})(x^2 + y^2 > 0)$

negace: $(\exists x, y \in \mathbb{R})(x^2 + y^2 \leq 0)$

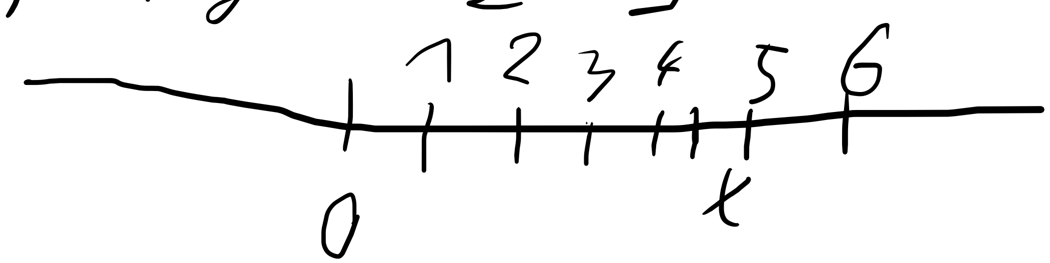
$$\boxed{\begin{aligned} \neg[(\forall x)\varphi(x)] &= (\exists x)(\neg \varphi(x)) \\ \neg[(\exists x)\varphi(x)] &= (\forall x)(\neg \varphi(x)) \end{aligned}}$$

$x = y = 0 \Rightarrow x^2 + y^2 = 0$

2. $(\forall x \in \mathbb{R})(\exists y \in \mathbb{N})[(y \leq x) \wedge (y+1) > x]$

$(\exists x \in \mathbb{R})(\forall y \in \mathbb{N})[(y > x) \vee (y+1) \leq x]$

$x = -8,2$



Dokažte matematickou indukcí

$$\sum_{i=1}^{n} (2i-1) = n^2$$ pro všechna $n \in \mathbb{N}$

Důkaz: 1) $n=1$: $\sum_{i=1}^{n} (2i-1) = \sum_{i=1}^{1} (2i-1) = 2-1 = 1 = n$

Tedy tvrzení platí pro $n=1$.

2) nechť tvrzení platí pro $n = k \in \mathbb{N}$.

Potom

$$\sum_{i=1}^{k+1} (2i-1) = \sum_{i=1}^{k} (2i-1) + (2(k+1)-1)$$

$$= k^2 + 2k + 1 = (k+1)^2$$

Tedy tvrzení platí pro $n = k+1$

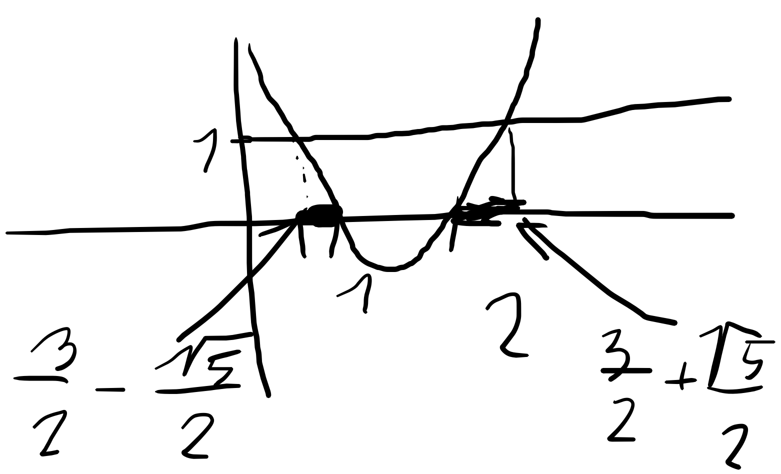
$\log_{\frac{1}{3}}(x^2-3x+2) \geq 0$

$\Updownarrow$

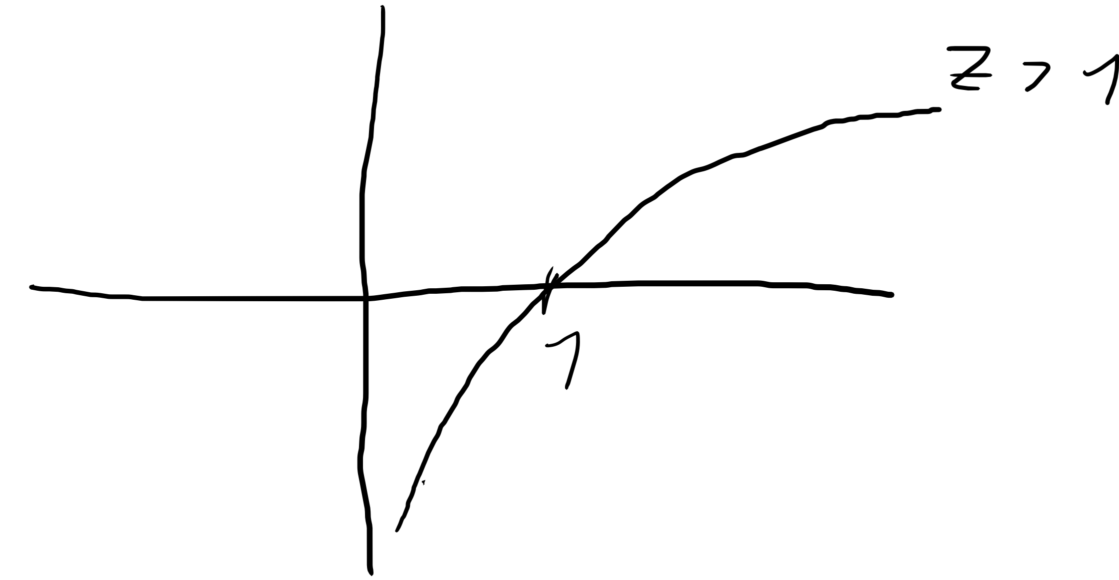
$x^2-3x+2 \in (0,1]$

$\parallel$

$(x-1)(x-2)$



$\log_z$



$\log_z(x) = a$

$\boxed{z^a = x}$

$e^{\log(x)} = x$

$z^{\log_z(x)} = x$

$x^2-3x+2 = 1$

$x^2-3x+1 = 0$

$D = 9-4 = 5$

$x_{1,2} = \frac{+3 \pm \sqrt{5}}{2} = \begin{cases} \frac{3}{2}+\frac{\sqrt{5}}{2} \\ \frac{3}{2}-\frac{\sqrt{5}}{2} \end{cases}$

řešení: $\left[\frac{3}{2}-\frac{\sqrt{5}}{2}, 1\right) \cup \left(2, \frac{3}{2}+\frac{\sqrt{5}}{2}\right]$

$$\neg(\varphi \Rightarrow \psi) = (\varphi \wedge \neg \psi)$$

Pokud prší, vezmu si deštník

prší $\Rightarrow$ nevezmu si deštník

φ	ψ	$\varphi \Rightarrow \psi$	$\neg(\varphi \Rightarrow \psi) = (\varphi \wedge \neg \psi)$
0	0	1	0
0	1	1	0
1	0	0	1
1	1	1	0

$$\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}, \quad n \in \mathbb{N}$$

Důkaz: ① $n=1$: $\sum_{i=1}^{n} i^2 = 1 = \frac{1(1+1)(2+1)}{6} = 1$ ✓

② nechť to platí pro $n = k$

$$\sum_{i=1}^{k+1} i^2 = \sum_{i=1}^{k} i^2 + (k+1)^2 = \frac{k(k+1)(2k+1)}{6} + (k+1)^2$$

$$(\text{chci}) = \frac{(k+1)(k+2)(2k+3)}{6} \quad \|$$

$$\frac{(k^2+3k+2)(2k+3)}{6} \qquad \frac{2k^3+3k^2+k+6k^2+12k+6}{6}$$

$$\| \qquad\qquad\qquad \|$$

$$\frac{2k^3+9k^2+13k+6}{6} = \frac{2k^3+9k^2+13k+6}{6}$$

HOTOVO

Tedy vzorec platí pro $n = k+1$

Dle matematické indukce vzorec platí pro všechna $n \in \mathbb{N}$