Permutation Groups and the Solution of German Enigma Cipher

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How do cipher systems work?

Őluİouèký kùò. The sender starts with a **plain text**. Then with the help of a **key** he produces a **cipher text**. The cipher text is created from the plain text and the key by an algorithm. This process is called **encryption**.

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An adversary may intercept the cipher text and attempt to recover the original plain text from it. This is called **solving the cipher**, informally **codebreaking**.

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Thus a cipher text usually has the form

```
indicator messagetext
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Enigma, first contacts

From 1928 the German army *Wehrmacht* started to test a new cipher system. This caused panic in the ranks of the Polish Secret Service. After the end of the First World War France and Great Britain believed that Germany was no longer a threat to their security. However, Poland never shared this opinion.

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Attempts to solve the new cipher had been completely unsuccessful for several years. Even parapsychology was involved but in vain. Finally, someone in the Polish Secret Service got the idea that mathematicians could be useful. A course in cryptanalysis was organized for students of mathematics at the University of Poznan.

Young Polish mathematicians

Three of the best graduates of the course,

Marian Rejewski (1905-1980),
Henryk Zygalski (1906-1978) and
Jerzy Rózycki (1907-1942)
then accepted the offer to work on cryptanalysis of the new cipher.

First results

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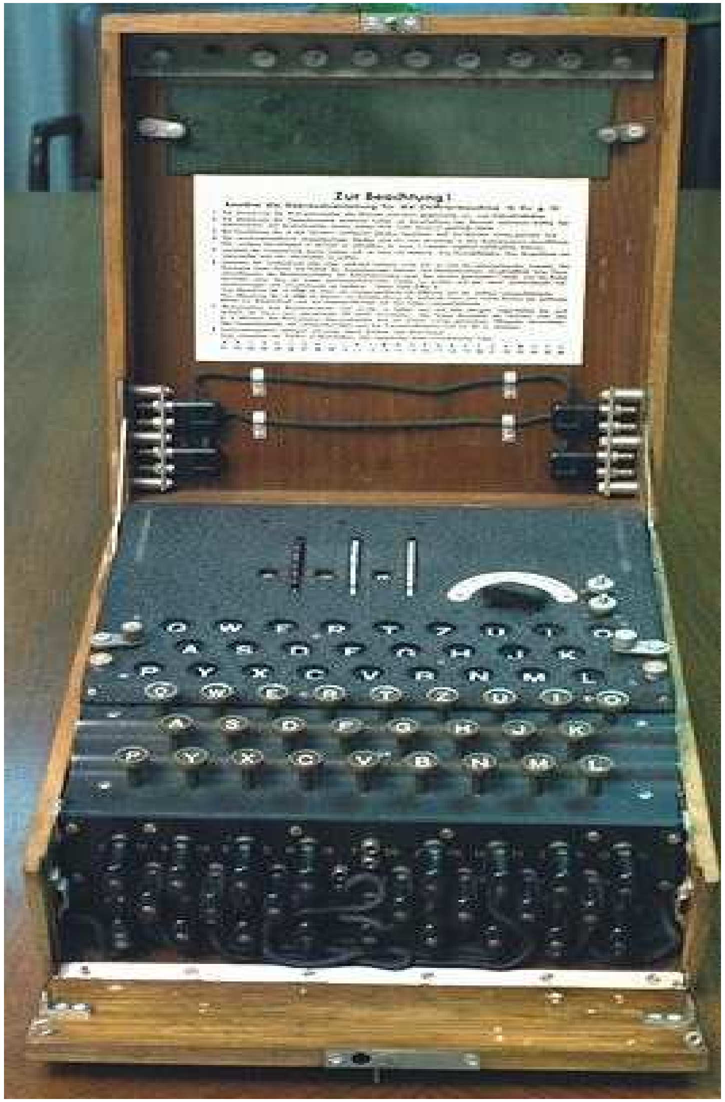
A similar cipher was produced by a commercial machine **Enigma** that had been on sale since 1926. So the Polish Intelligence Service hypothesized that the new cipher was produced by a military version of the **Enigma** machine.

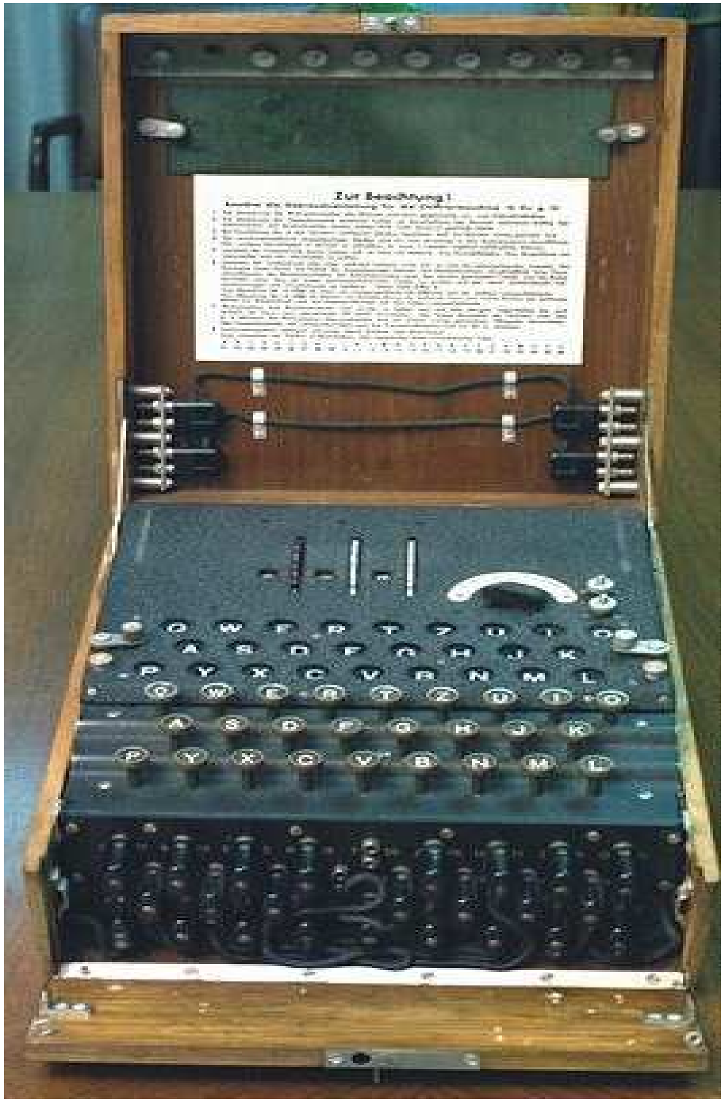
Military Enigma



Important parts of the machine are

- keyboard,

Military Enigma



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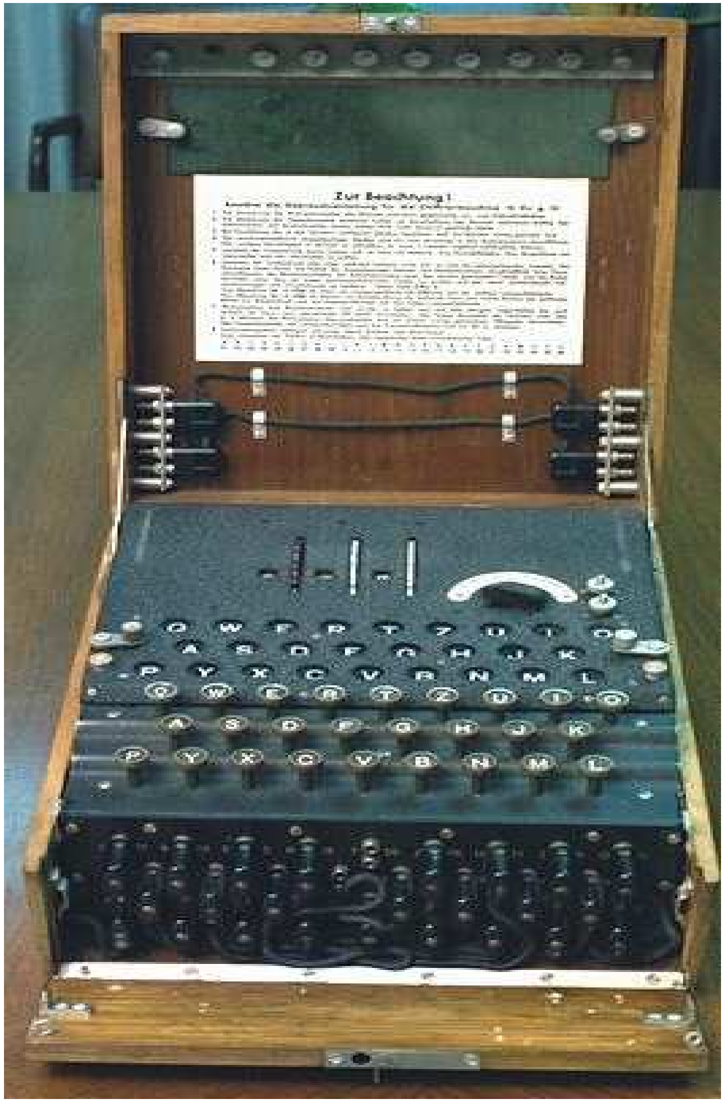

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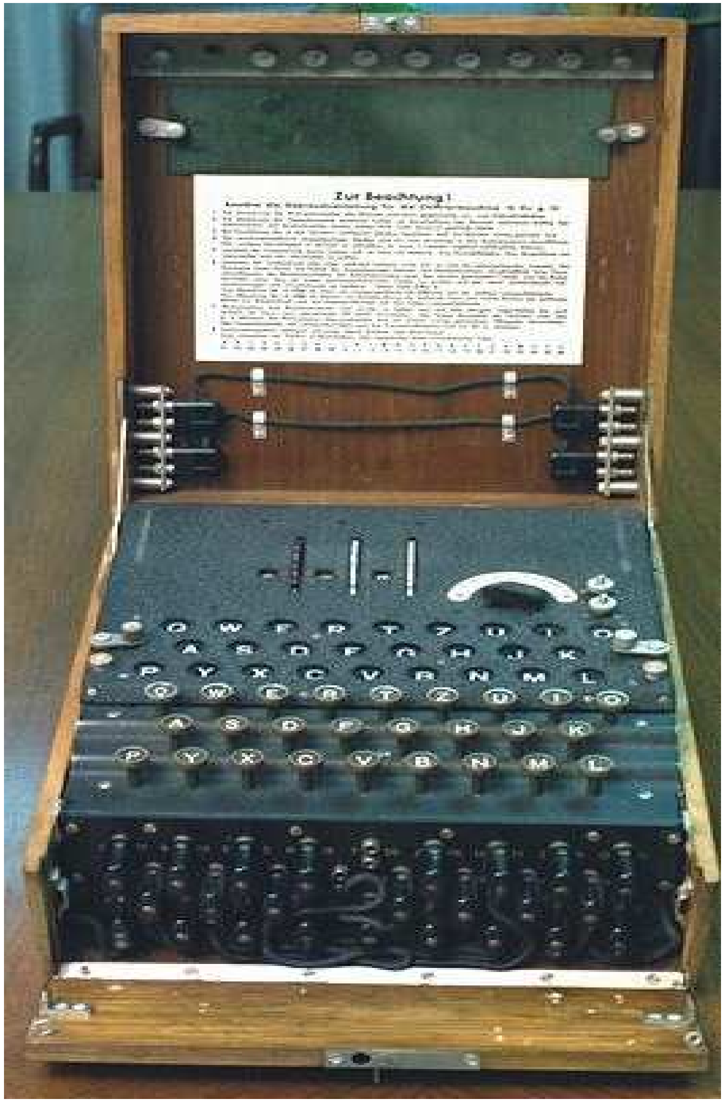

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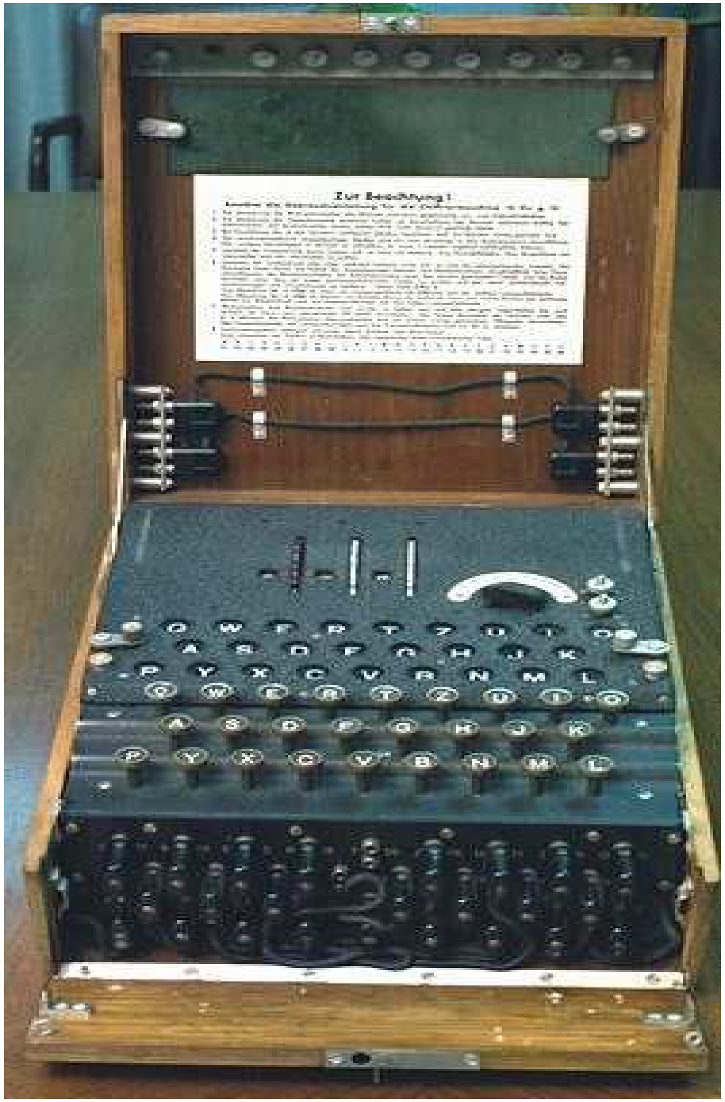

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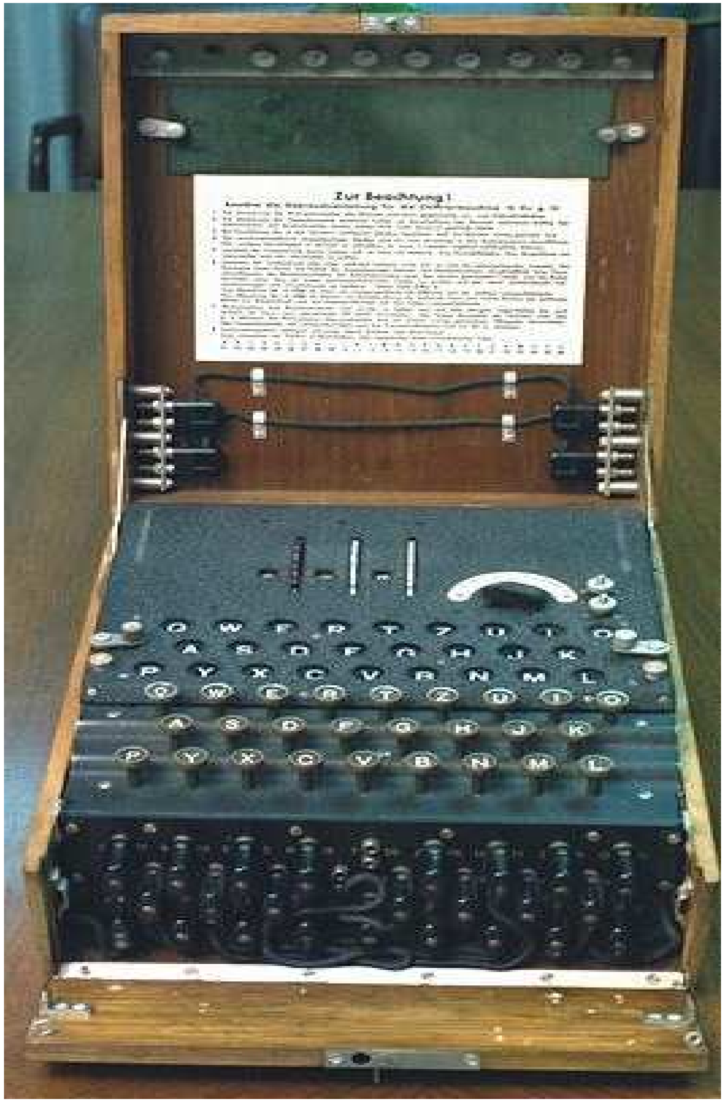

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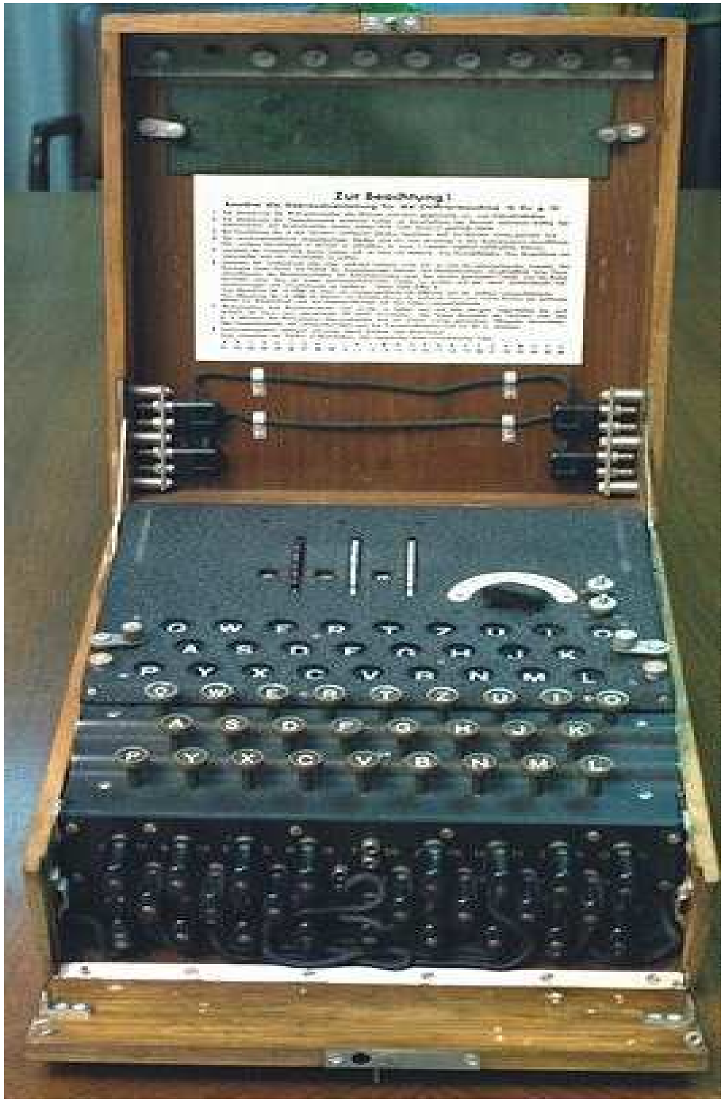

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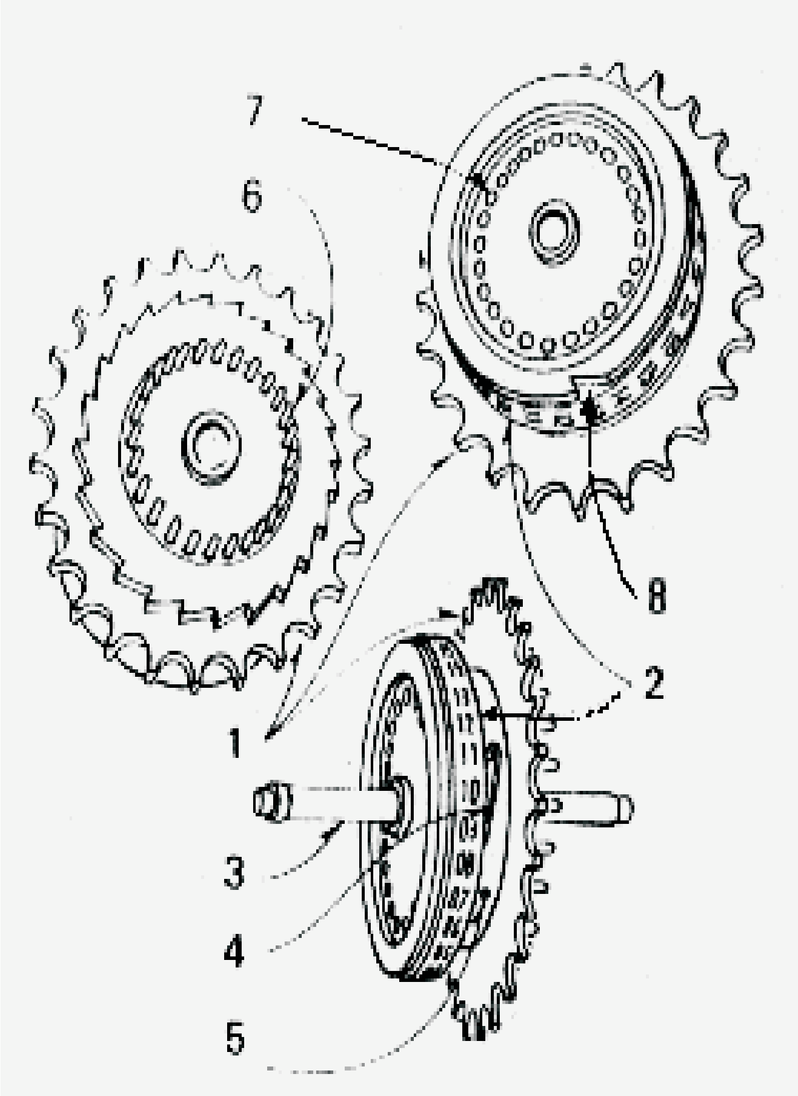
- rotor,
- reflector (Umkehrwalze).

Rotors



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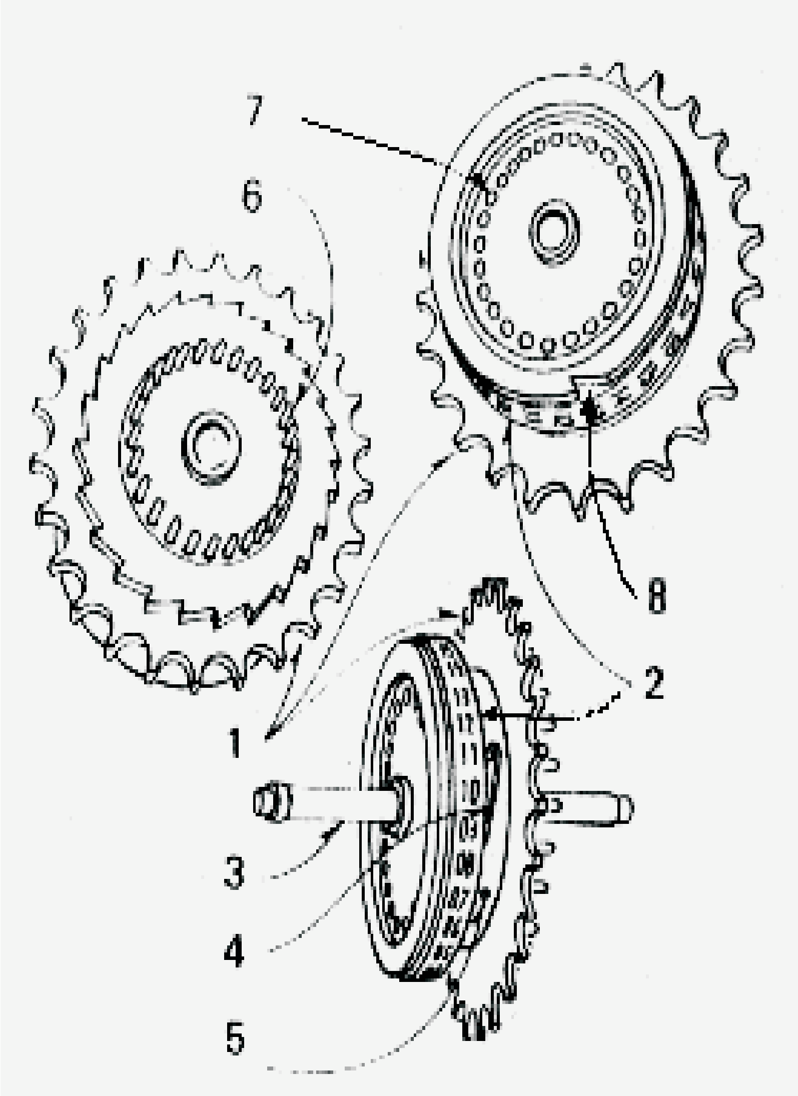

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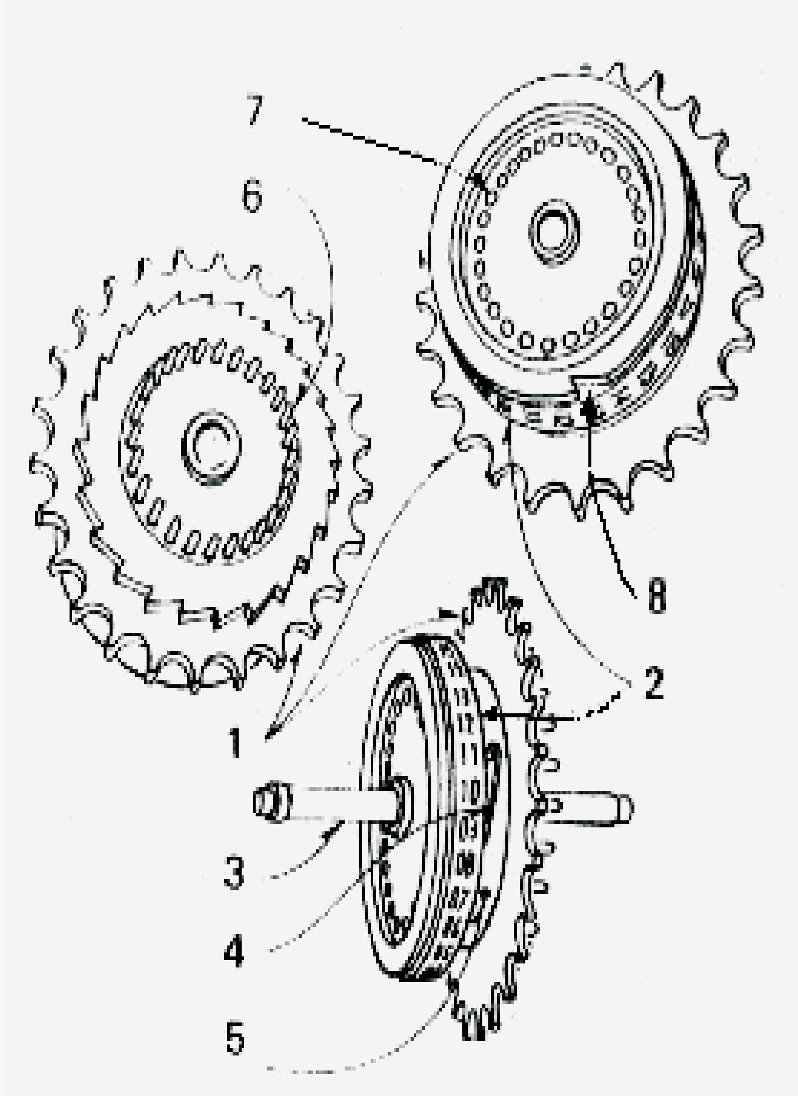

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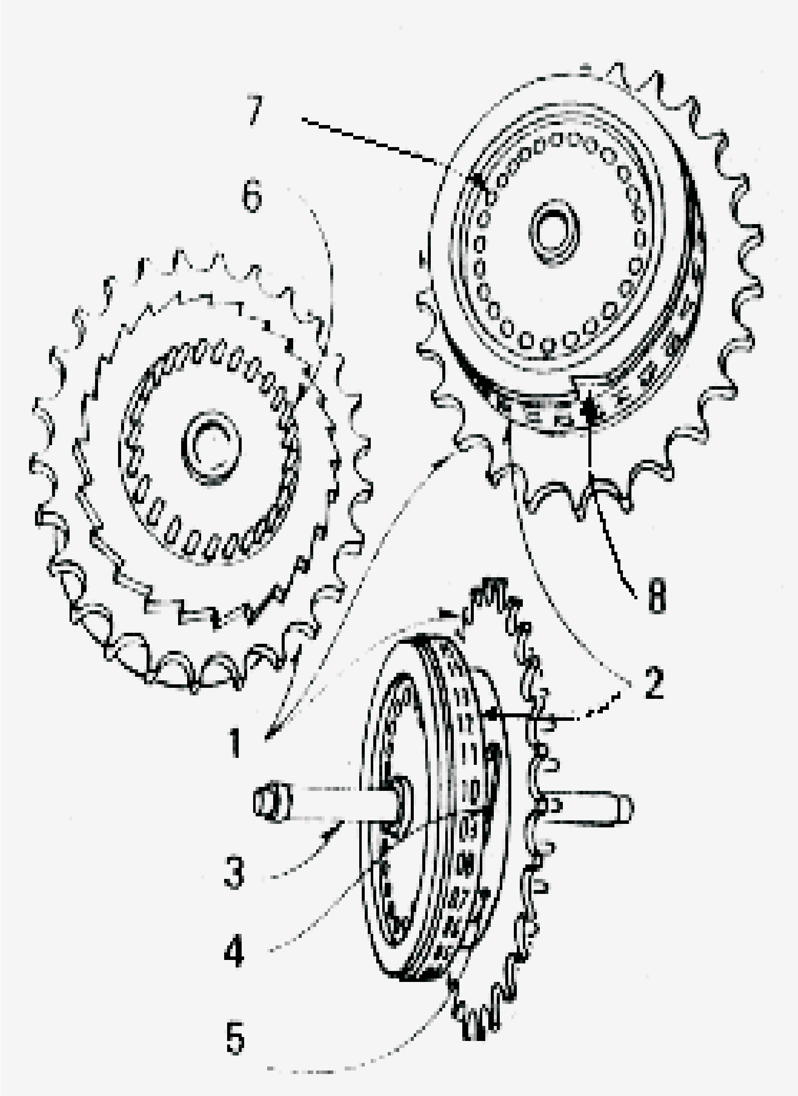

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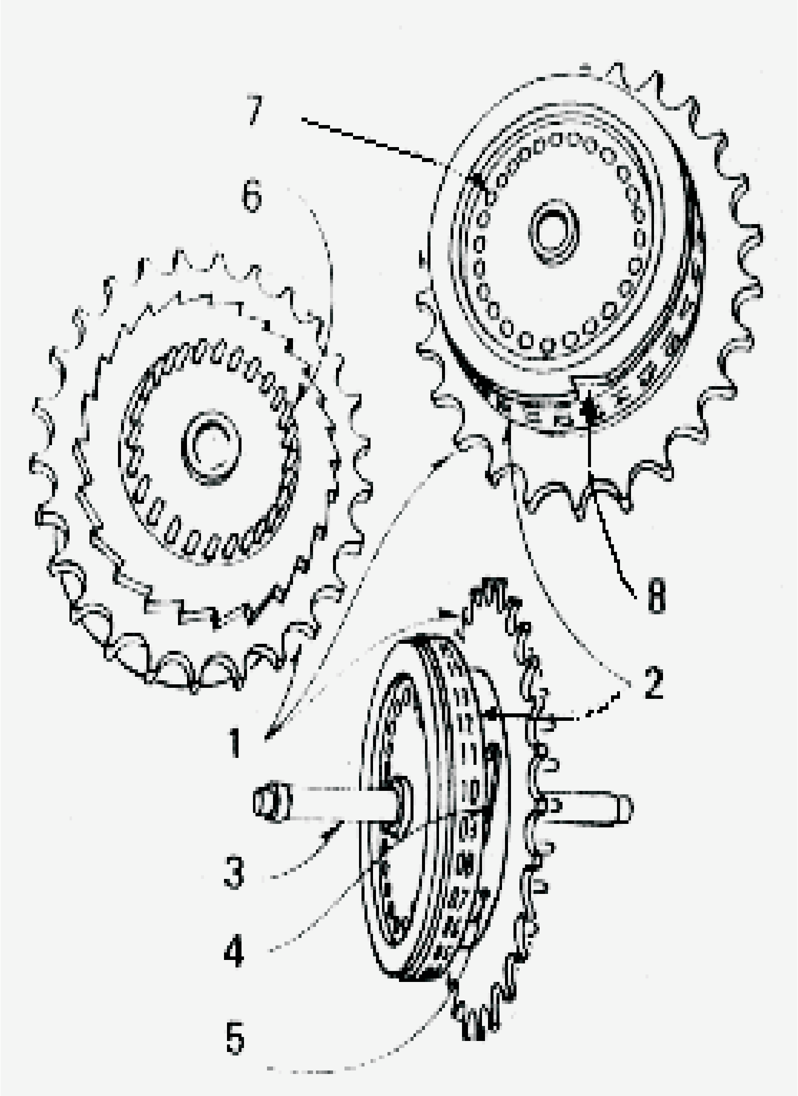

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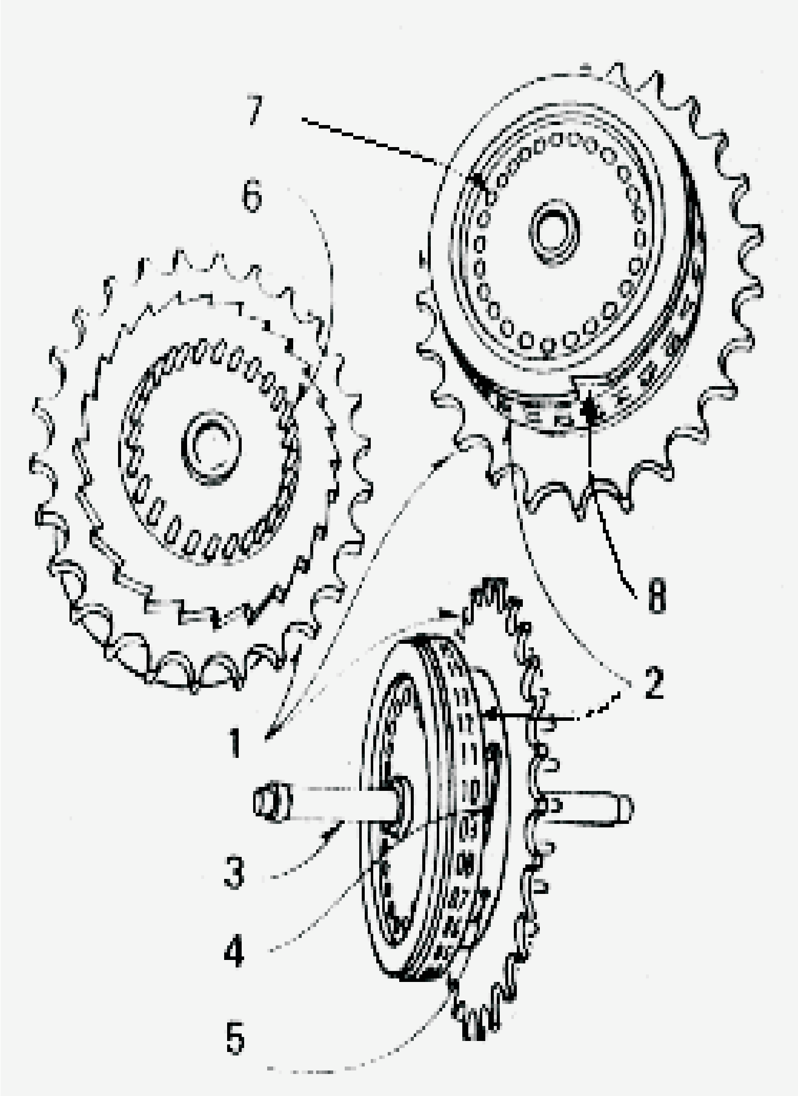

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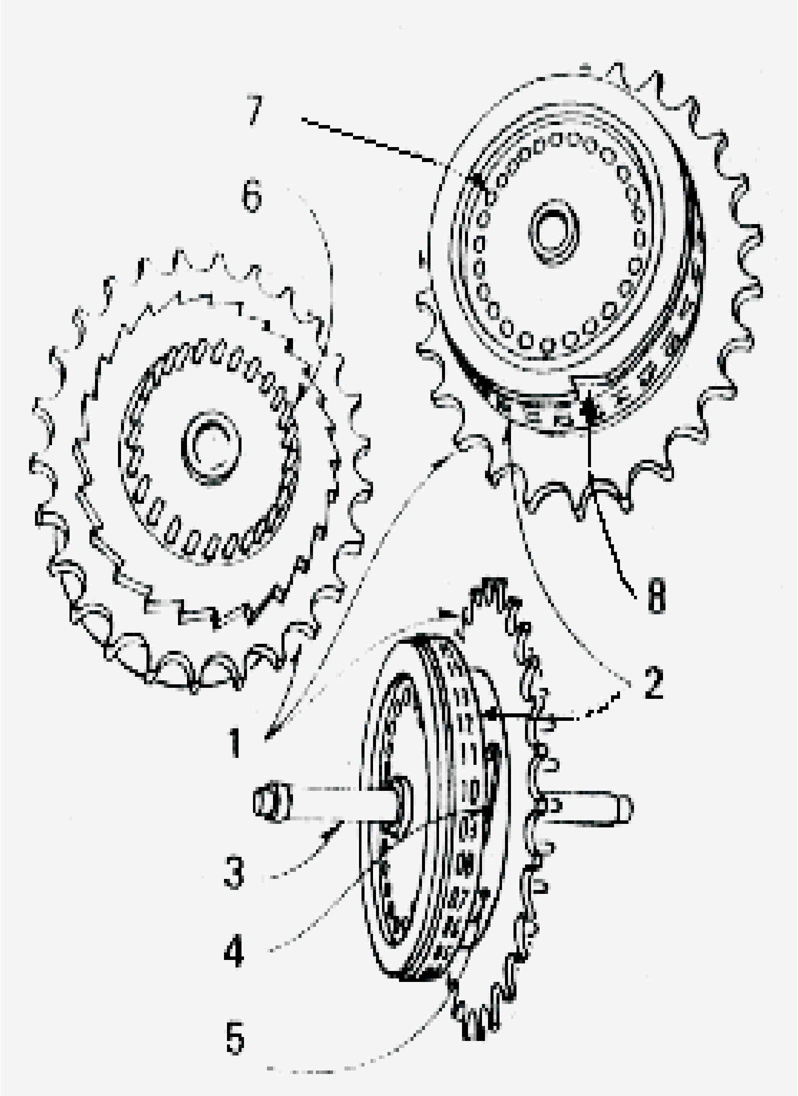

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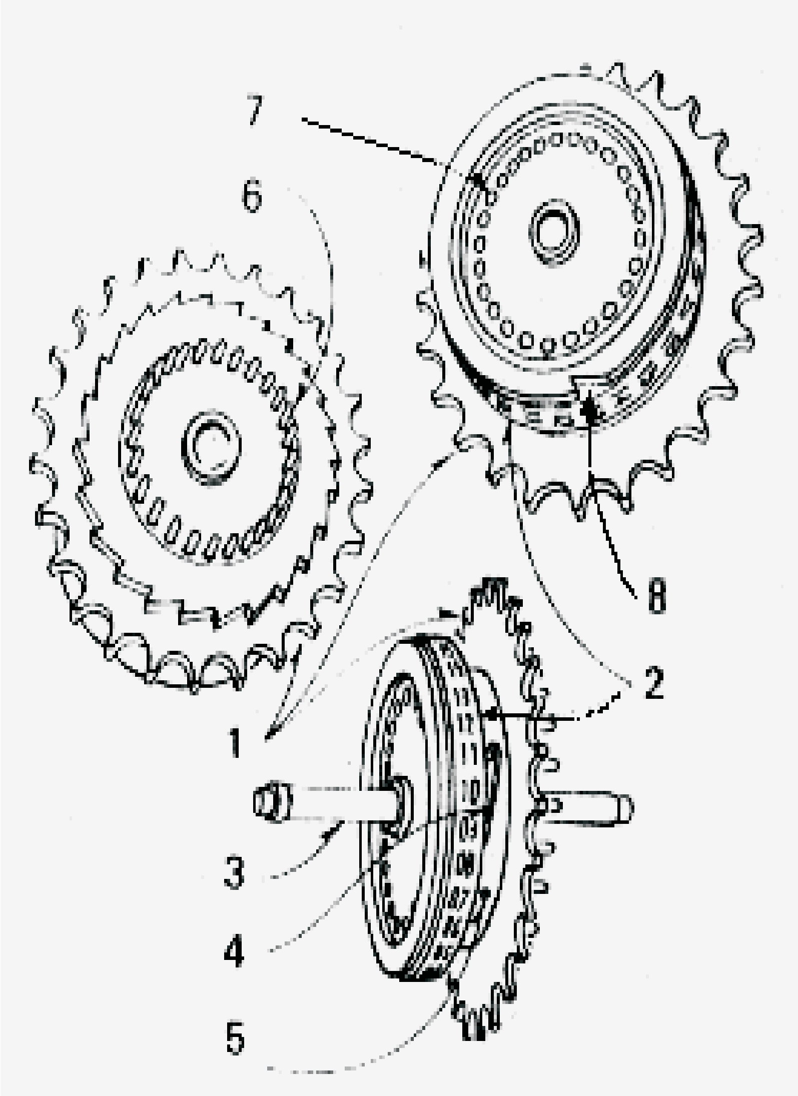

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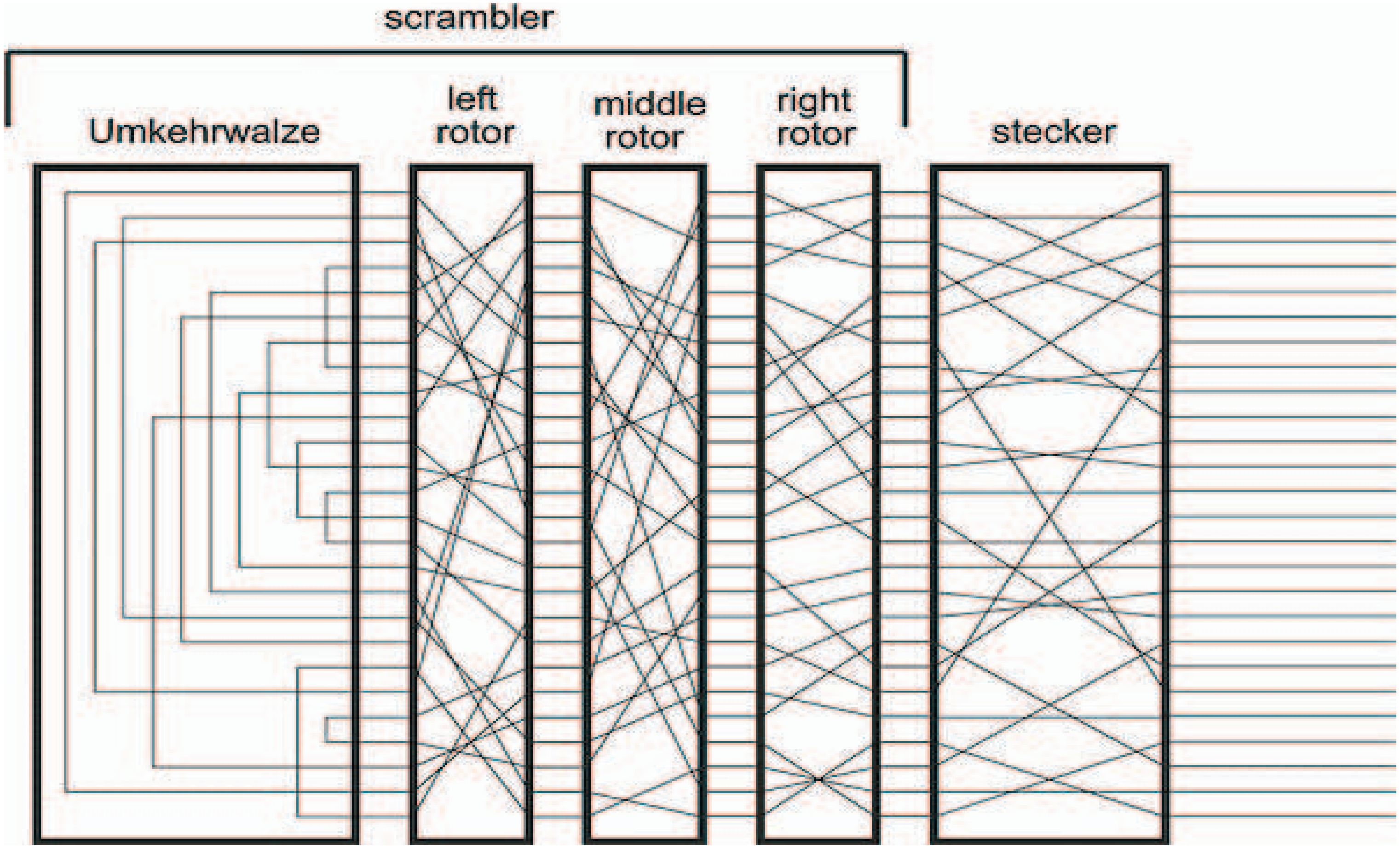

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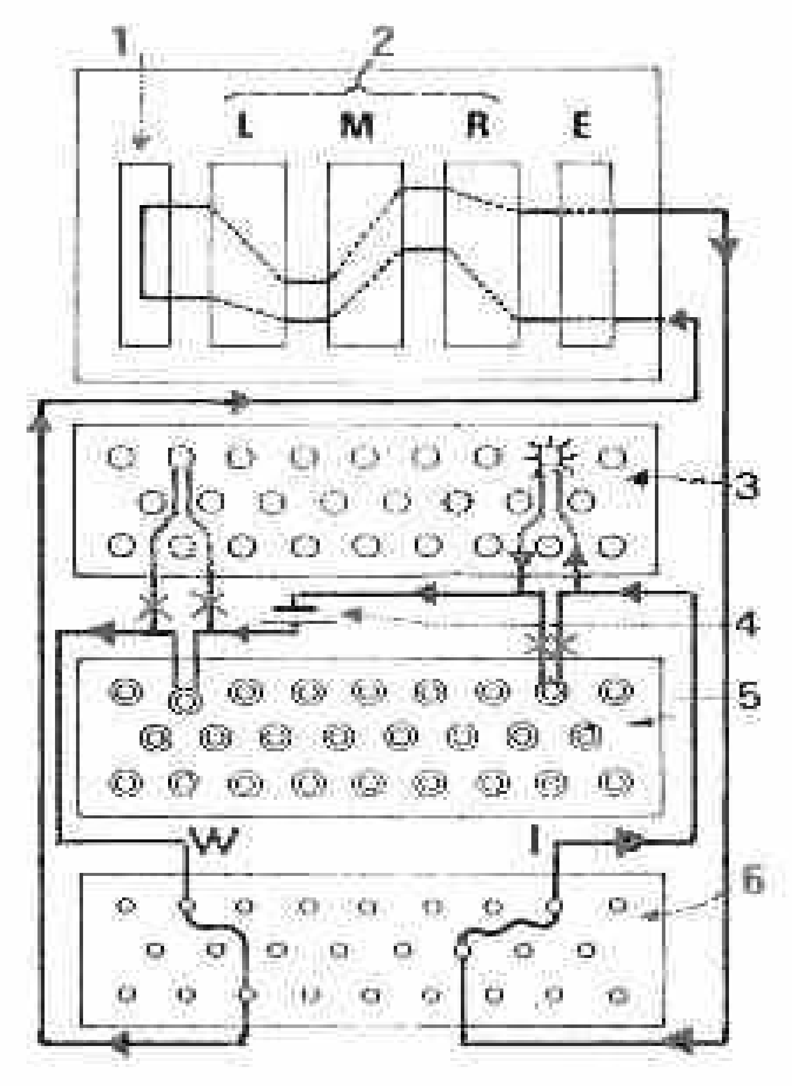

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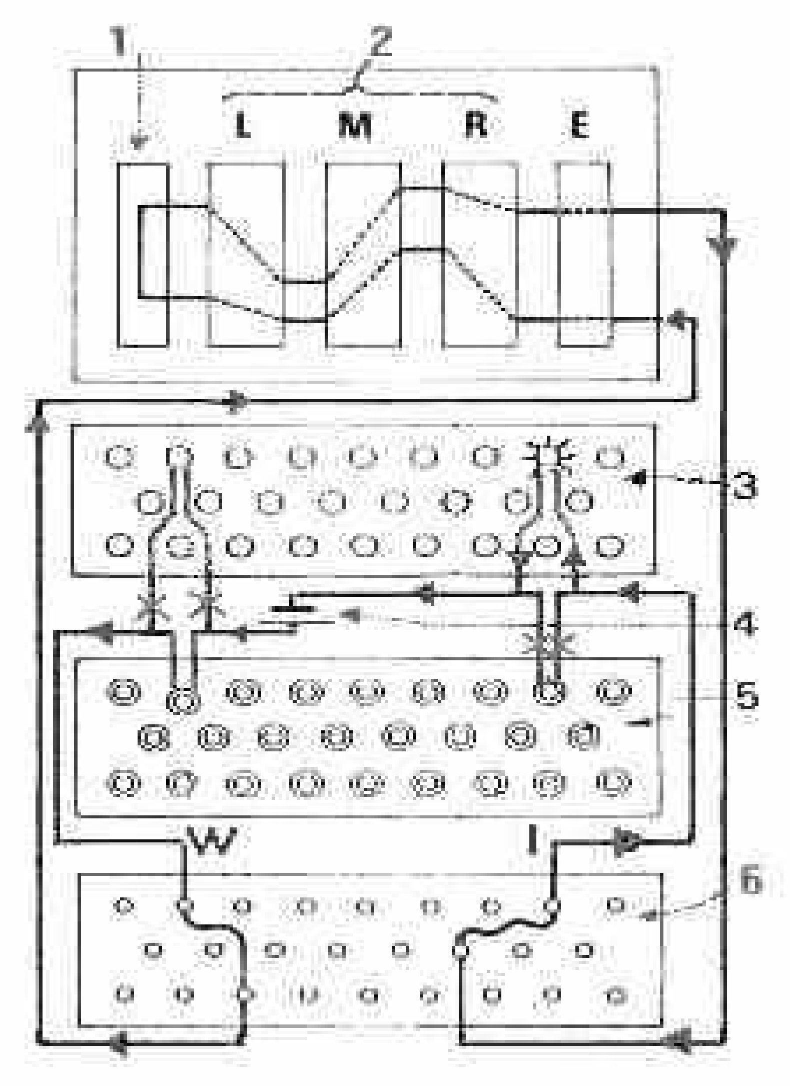

This is how the current flows through the Enigma machine after pressing a key.

Operation manual

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the basic setting, letters visible in the little windows: e.g. `UFW`.

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Thus the message `HELLO` was encrypted as `BPTQS`.

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The violation of two cryptological maxims was the starting point of a mathematical analysis of the Enigma cipher. Could this open the door to solve the cipher?

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and several dozens intercepted encrypted messages from each day of these two months.

Permutations

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The value of a permutation P at a point x will be written as xP. Every permutation P on a set X uniquely defines the **inverse permutation** P^{-1}. This is determined by the property

$$(xP)P^{-1} = x$$

for every $x \in X$.

Product of permutations

Any two permutations P, Q on the same set X can be composed (as mappings) to get the **composition** or **product** PQ of the two permutations. Its value at a given element $x \in X$ is

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Thus $PP^{-1} = I = P^{-1}P$ for every permutation P on X.

Graph of a permutation

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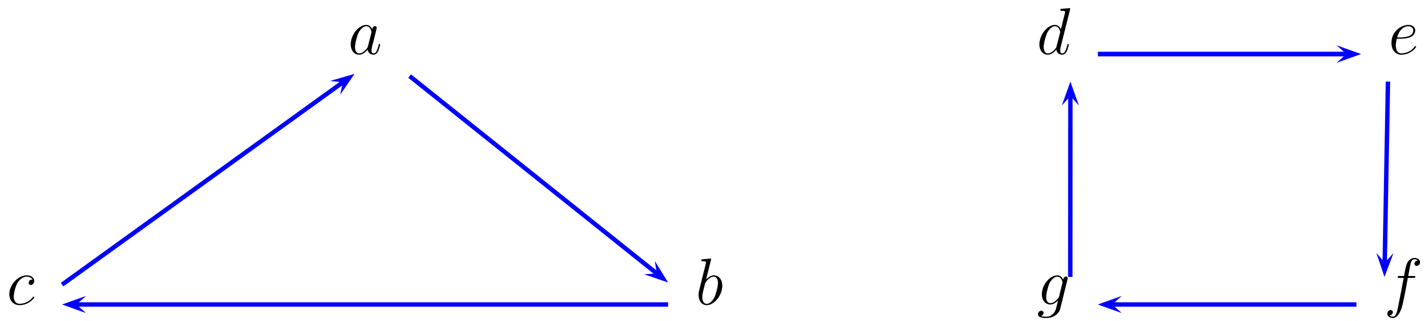
For example, the permutation P on the set $\{a, b, c, d, e, f, g\}$ defined by

$$aP = b, bP = c, cP = a, dP = e, eP = f, fP = g, gP = d,$$

can be visualized as



Graph of the composition

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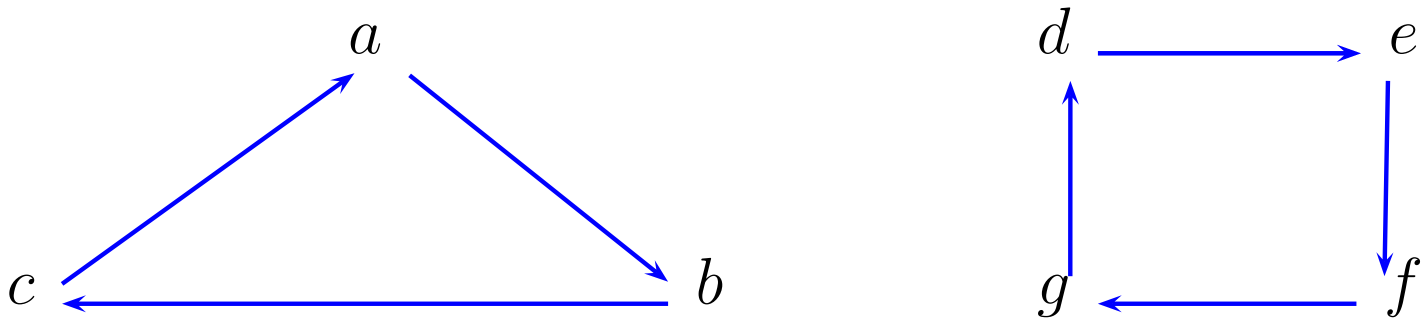

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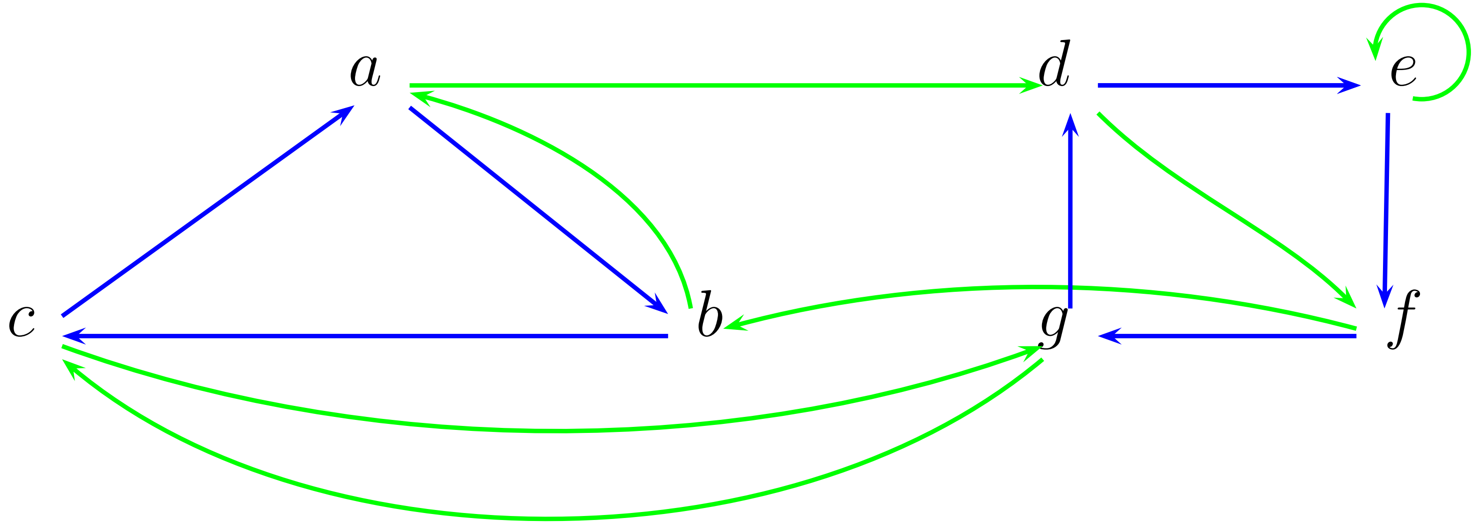
Then we draw the graph of Q to it.



Graph of the composition - cont.

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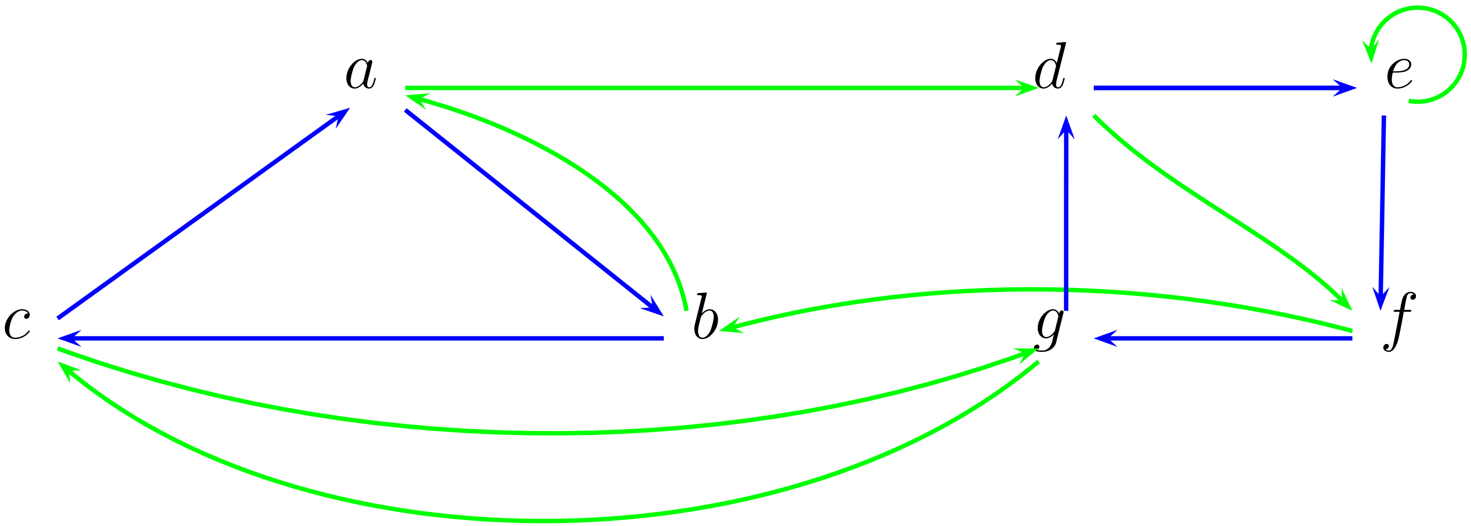

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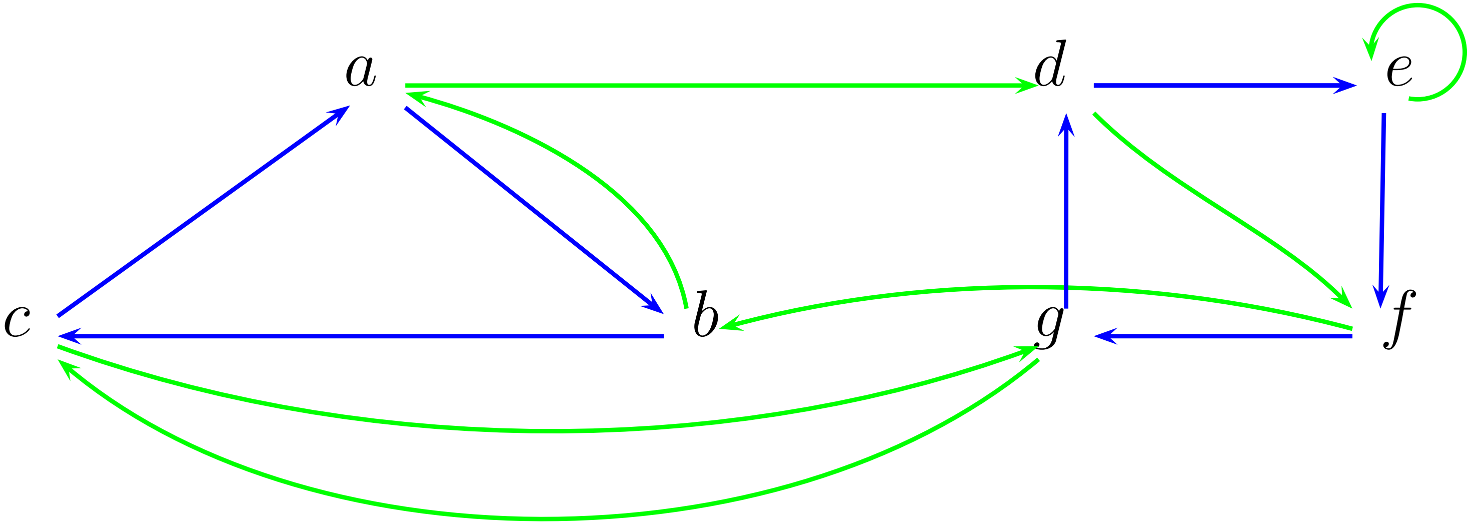

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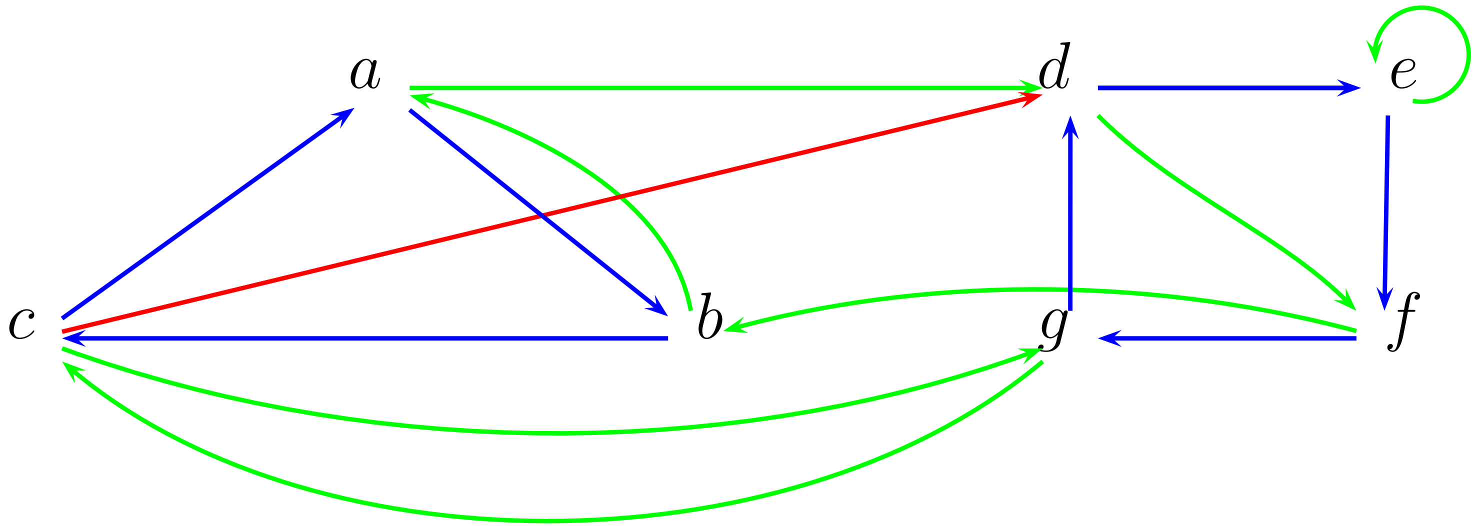

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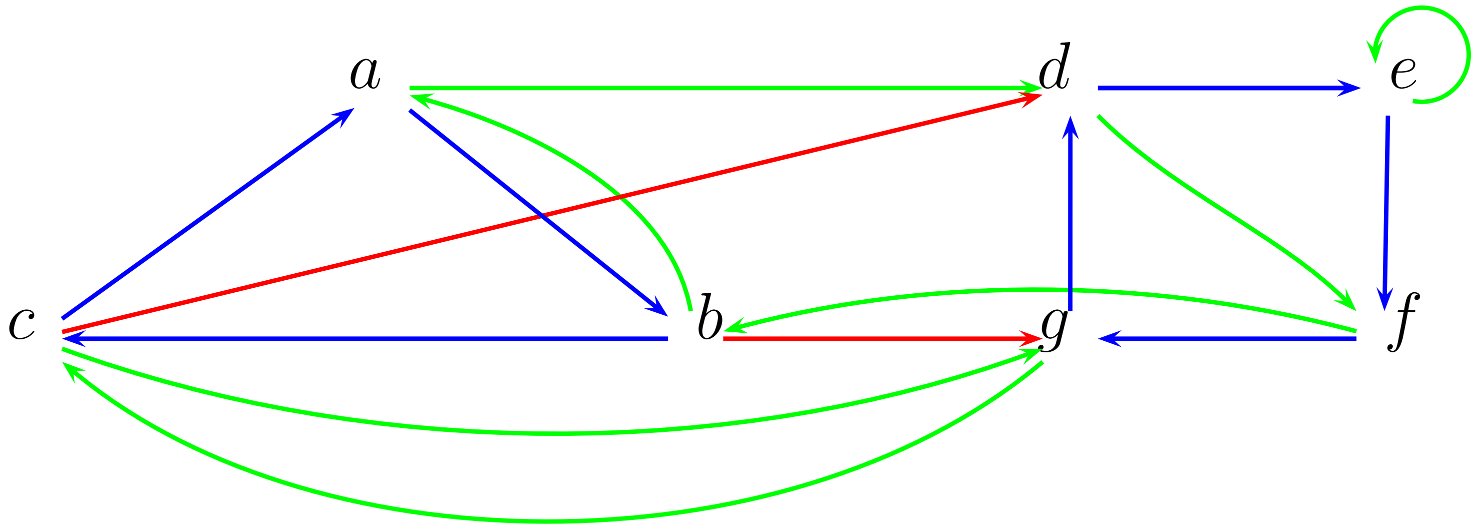

Math. model of rotors and reflector

If we take a rotor, we may denote the spring contacts on one side of the rotor by letters of the alphabet $a, b, c, \ldots, x, y, z$. Opposite to each spring contact there is a disc, through which the current flows out of the rotor. We denote it by the same letter as the opposite spring contact. Thus the **wires inside the rotor define** a one-to-one mapping, or **a permutation**, on the alphabet $\{a, b, c, \ldots, x, y, z\}$.

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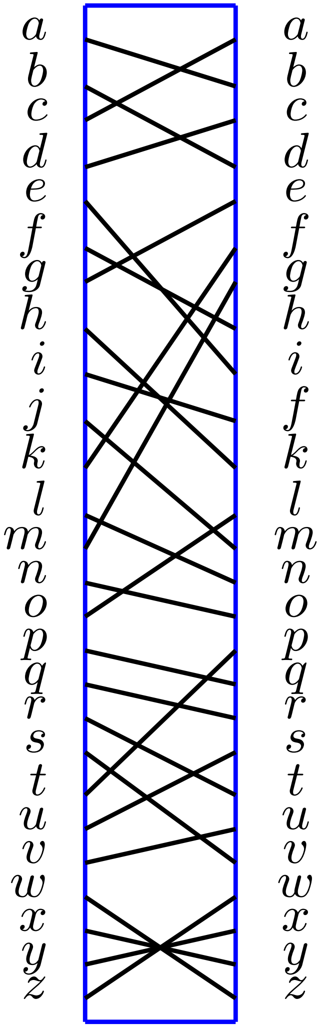
There are also thirteen wires inside the reflector, each connecting two springs. Thus the wiring inside the **reflector can be described by** another **permutation** R on the alphabet $\{a, b, c, \ldots, x, y, z\}$. This time the permutation R is not arbitrary but has to have 13 cycles of length 2.

Math. model of a rotor


a b c d e f g h i j k l m n o p q r s t u v w x y z
a b c d e f g h i f k l m n o p q r s t u v w x y z

If we take a rotor, we may denote the spring contacts on one side of the rotor by the letters of the alphabet $a, b, c, \ldots, x, y, z$. Opposite to each spring contact there is a disc, through which the current flows out of the rotor. We denote it by the same letter as the opposite spring contact. Thus the wires inside the rotor define a permutation on the alphabet $\{a, b, c, \ldots, x, y, z\}$.

More permutations

Let us denote the permutations that describe the wirings of the left, middle and right rotors by L, M and N.

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And the wires between the plugboard and the entry wheel define yet another permutation H on the alphabet $\{a, b, c, \ldots, x, y, z\}$.

Static model of the scrambler

The passage of the electrical current through the scrambler now can be described as the composition of permutations

$$NMLRL^{-1}M^{-1}N^{-1}.$$

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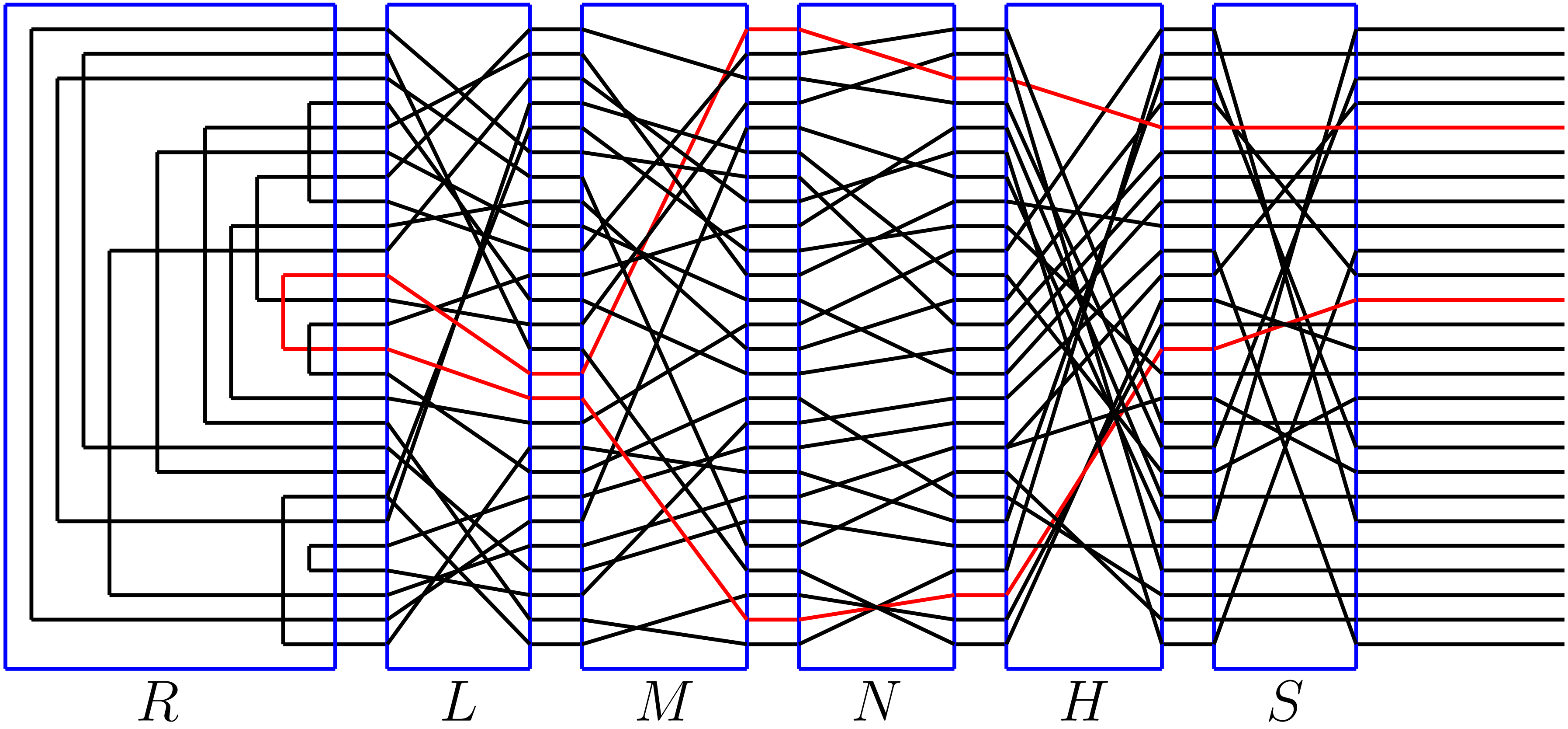
And we also have to take into account the cables in the plugboard defining a permutation S.

Static model of Enigma



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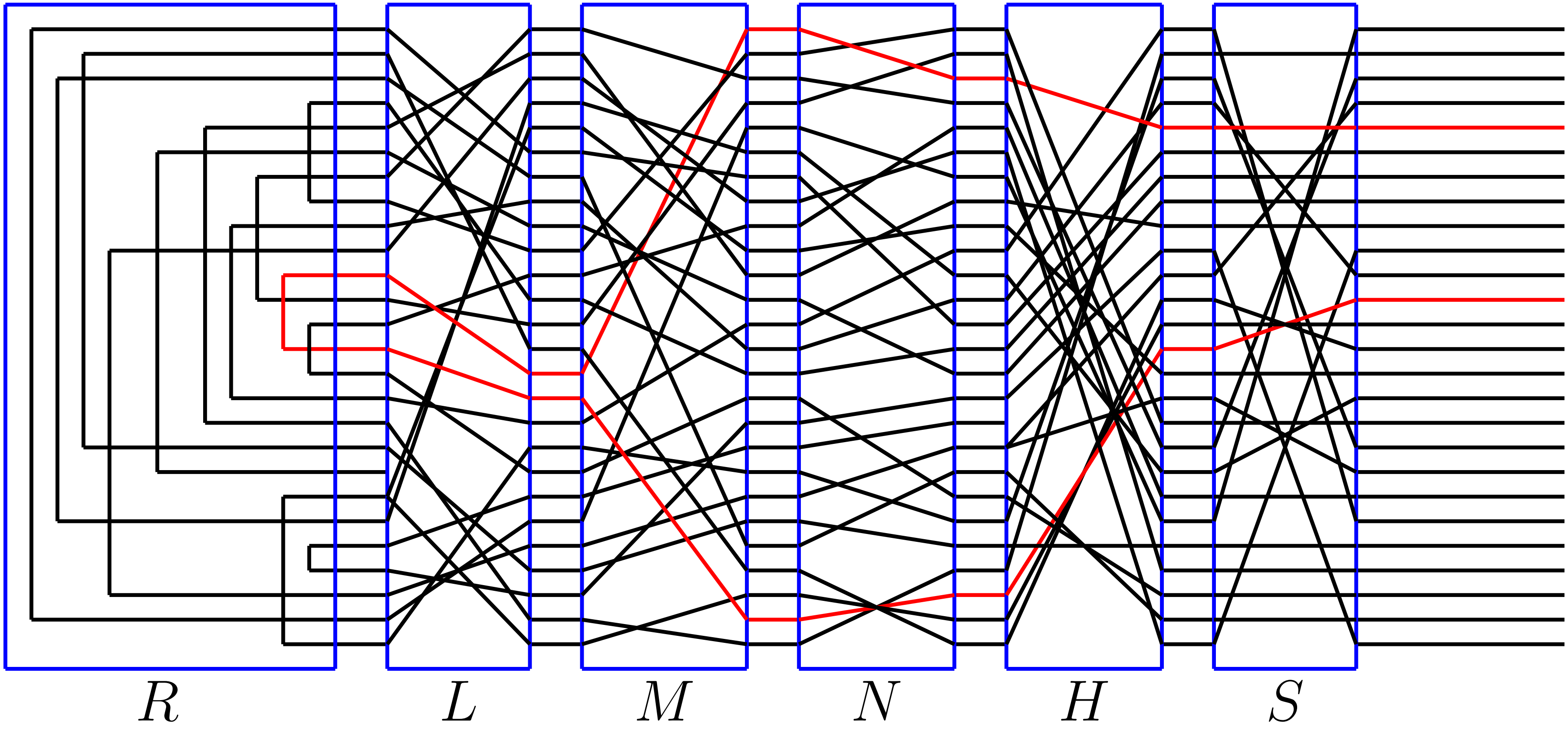

$$SHNMLRL^{-1}M^{-1}N^{-1}H^{-1}S^{-1}$$

Dynamic model of the scrambler

If the right rotor moves first, then the current from the disc a of the entry wheel does not flow to the spring a of the right rotor, but to the spring b of the right rotor. Similarly, from the disc b of the entry wheel it flows to the spring c of the right rotor, etc.

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We denote by P the cyclic permutation

$$\begin{tikzcd}
a \arrow[r] & b \arrow[r] & c \arrow[r] & d & \cdots & x \arrow[r] & y \arrow[r] & z \arrow[lllllll, bend left]
\end{tikzcd}$$

It maps every letter of the alphabet $\{a, b, c, \ldots, x, y, z\}$ to the subsequent one and the last letter z to the first letter a.

Dynamic model of the scrambler

But we have to take into account also the fact that after pressing a key the right rotor moves first and only then the current flows through the plugboard, entry wheel and the scrambler. So the current from the disc a of the entry wheel does not flow to the spring a of the right rotor, but to the spring b of the right rotor. Similarly, from the disc b of the entry wheel it flows to the spring c of the right rotor, etc.

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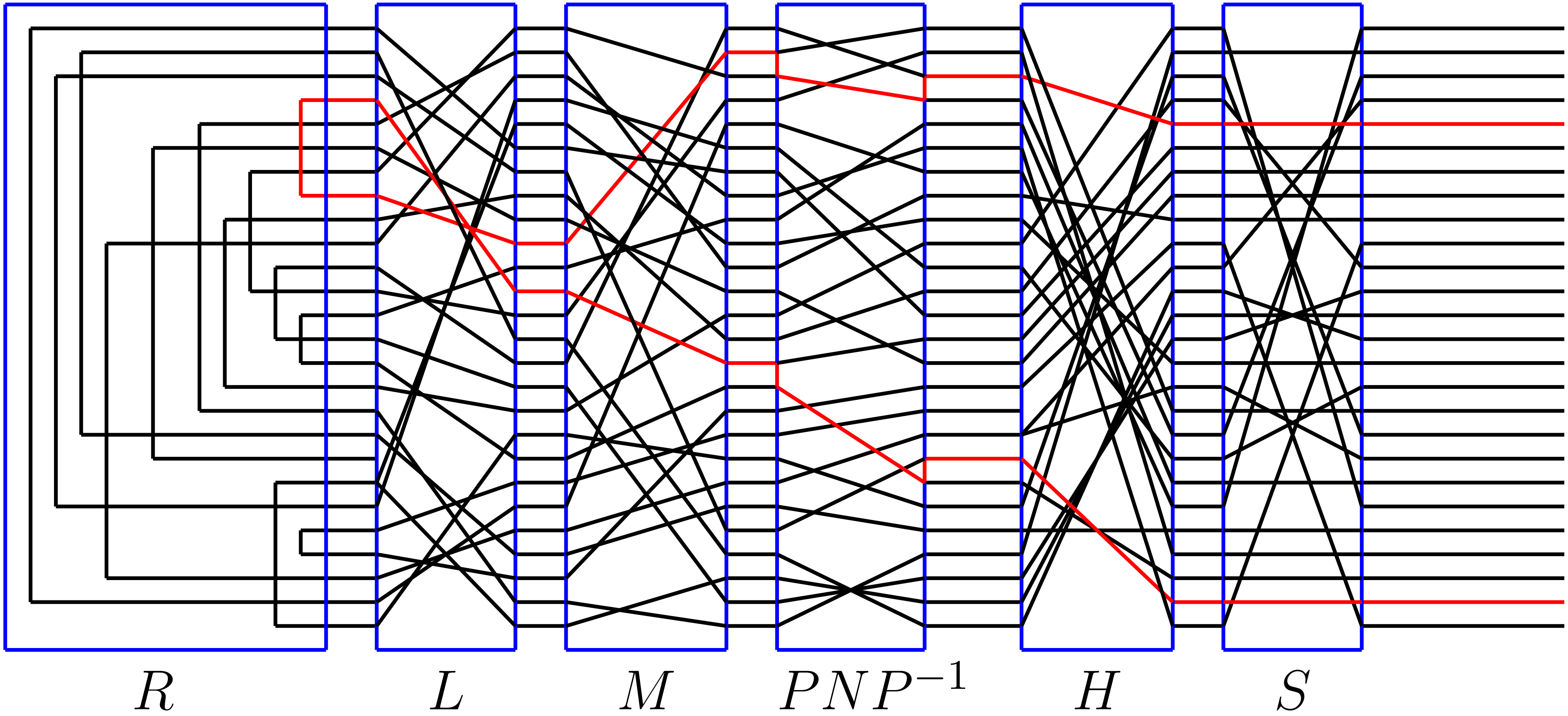
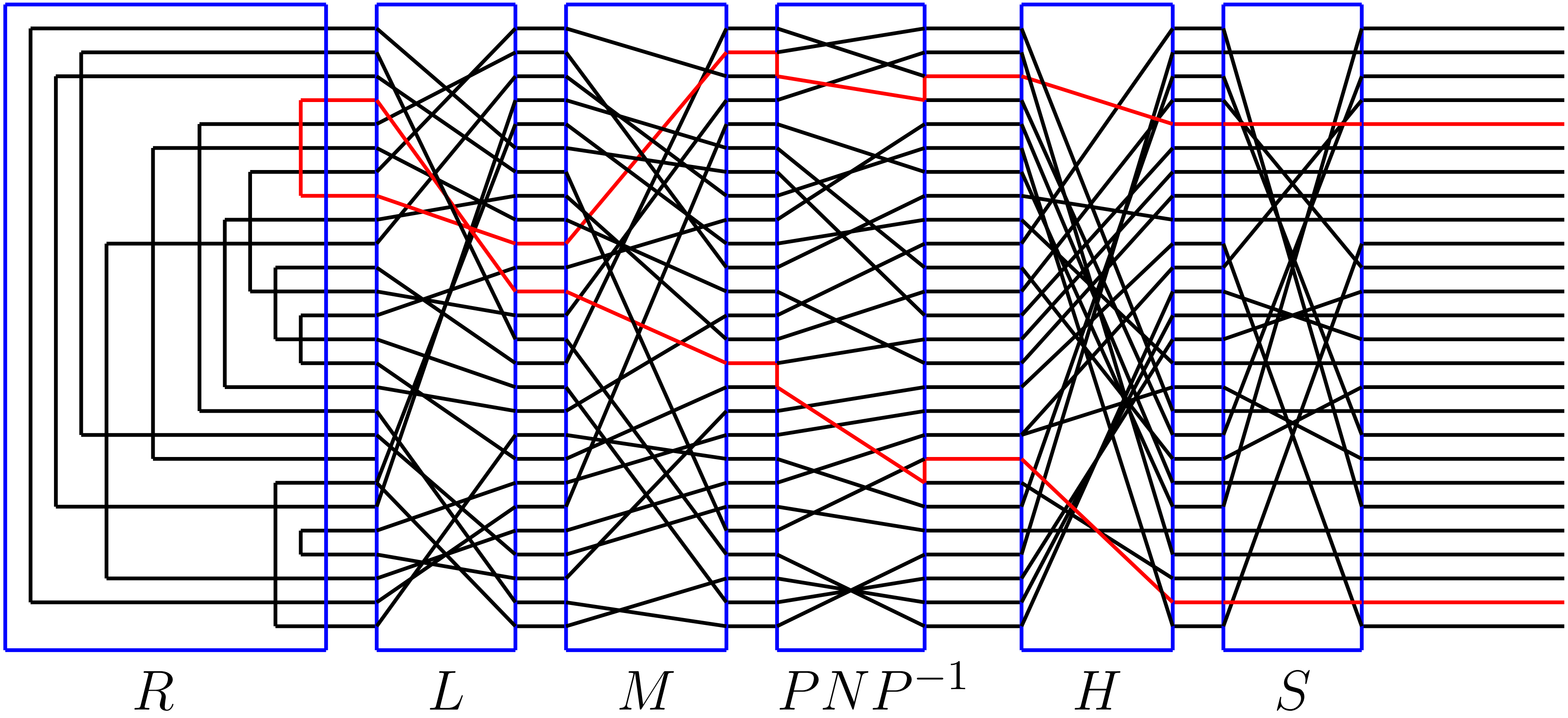
Dynamic model of Enigma



Dynamic model of Enigma



$$SH(PNP^{-1})MLRL^{-1}M^{-1}(PN^{-1}P^{-1})H^{-1}S^{-1}$$

Complete model

Thus the whole dynamic model of the operation of the Enigma machine can be described by the permutation

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The cyclic permutation P is known, the unknown permutations L, M, N and H describe the unknown internal structure of the Enigma machine. The permutation S changes day by day and is given by the corresponding daily key.

Permutations of the day

The first six letters of each message transmitted during the same day were encrypted by the same key given by the setup of the machine for that day. So we can denote by A the permutation of the first letters of messages transmitted that day.

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Similarly, we denote by B the permutation of the second letters of messages transmitted the same day, by C the permutation of the third letters, by D, E and F the permutations of the fourth, fifth and sixth letters.

All the six permutations A, B, C, D, E, F were also unknown. We may call them the **permutations of the day**.

Connection to the dynamic model

We have already found another description of the permutation determined by the setup of the machine for the day. It was

$$SHPNP^{-1}MLRL^{-1}M^{-1}PN^{-1}P^{-1}H^{-1}S^{-1}.$$

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$$SHPNP^{-1}MLRL^{-1}M^{-1}PN^{-1}P^{-1}H^{-1}S^{-1}.$$

So we get the equation

$$A = SHPNP^{-1}MLRL^{-1}M^{-1}PN^{-1}P^{-1}H^{-1}S^{-1}.$$

Connection cont.

We can find a similar expression for the second permutation of the day B. We only have to take into account that after pressing the second key the right rotor has already turned twice. So the equation is

$$B = SHP^2NP^{-2}MLRL^{-1}M^{-1}P^2N^{-1}P^{-2}H^{-1}S^{-1}.$$

Connection cont.

We can find a similar expression for the second permutation of the day B. We only have to take into account that after pressing the second key the right rotor has already turned twice. So the equation is

$$B = SHP^2NP^{-2}MLRL^{-1}M^{-1}P^2N^{-1}P^{-2}H^{-1}S^{-1}.$$

The remaining four permutations of the day can be expressed as

$$\begin{aligned}
C &= SHP^3NP^{-3}MLRL^{-1}M^{-1}P^3N^{-1}P^{-3}H^{-1}S^{-1},\\
D &= SHP^4NP^{-4}MLRL^{-1}M^{-1}P^4N^{-1}P^{-4}H^{-1}S^{-1},\\
E &= SHP^5NP^{-5}MLRL^{-1}M^{-1}P^5N^{-1}P^{-5}H^{-1}S^{-1},\\
F &= SHP^6NP^{-6}MLRL^{-1}M^{-1}P^6N^{-1}P^{-6}H^{-1}S^{-1}.
\end{aligned}$$

products

Conjugated permutations

It should be emphasized that these equations are valid only under the assumption that the only rotor that moved during the encryption of the six letters of the message keys during the given day was the right one. But this happened on average in 20 out of 26 days. Quite often.

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All the permutations in the previous expressions of the permutations of the day were unknown except P. But something important was known!

Before we proceed let us state an important definition.

Definition. Two permutations K, L on the same set X are called **conjugated** if there exists another permutation T on the set X such that

$$K = TLT^{-1}.$$

The theorem that won WWII

And one more definition.

Definition. The list of lengths of all cycles in a permutation K is called the **cyclic structure** of the permutation K.

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Definition. The list of lengths of all cycles in a permutation K is called the **cyclic structure** of the permutation K.

Theorem. Two permutations K, L on the same set X are conjugated if and only if they have the same cyclic structure.

We can get an idea why the theorem is true by drawing the graphs of permutations. So assume that permutations K, L are conjugated and let T be a permutation such that

$$K = TLT^{-1}.$$

Proof

Now choose an arbitrary element $x \in X$ and look at the following part of the graphs of the three permutations. The arrows of the permutation K are blue, the arrows of L are green, and the arrows of T are red.

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$$\begin{tikzcd}
x \arrow[r] \arrow[d] & xT \arrow[d] \\
xK \arrow[r] & xTL = xKT
\end{tikzcd}$$

Proof cont.

Thus the permutation T maps each cycle of the permutation K to a cycle of the permutation L of the same length. Conjugated permutations must have the same cyclic structures.

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Now assume conversely that two permutations K, L have the same cyclic structure. Choose a cycle in the permutation K and a cycle in the permutation L of the same length. Further, choose an element x in the chosen cycle of K and an element y in the chosen cycle of the permutation L. Try to set $xT = y$.

Proof cont.

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We search for a permutation T satisfying the equation

$$K = TLT^{-1}.$$

Proof cont.

Look at the following part of the graphs of all three permutations.

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$$\begin{tikzcd}
x \arrow[r] \arrow[d] & y = xT \arrow[d] \\
xK \arrow[r, dashed] \arrow[d] & yL \arrow[d] \\
xK^2 \arrow[r, dashed] \arrow[uu, dotted, bend left] & yL^2 \arrow[uu, dotted, bend left]
\end{tikzcd}$$

End of the proof

Thus the choice of x and y uniquely determines the values of the permutation T at all elements of the chosen cycle of the permutation K.

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Observe also that this method enables us to find all possible permutations T that satisfy the equation

$$K = TLT^{-1}$$

if the given permutations K, L have the same cyclic structure.

Characteristics of the day

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All permutations A, B, C, D, E, F of the day are conjugated to R. So they all have only cycles of length 2. Thus we get

$$A^2 = B^2 = C^2 = D^2 = E^2 = F^2 = I,$$

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or stated otherwise, each of the six permutations is equal to its inverse.

We do not not the permutations A, B, C, D, E, F yet. But we do know the products AD, BE, CF if there are enough intercepted messages for the day. Rejewski called them the **characteristics of the day**.

Finding characteristics

You will recall that the intercepted indicators were obtained by enciphering message keys twice. Stated otherwise, by enciphering messages of the form

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where x, y, z can be arbitrary letters.

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since $A^{-1} = A$.

But various pairs of the letters $u, v = uAD$ are known! They are the first and fourth letters of the intercepted messages.

A busy manoeuver day

So if there are enough intercepted messages from a given day, we do know the characteristics AD, BE and CF of the day.

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As an example, we find the characteristics of a busy manoeuver day, when the following indicators were intercepted. This is a made up example taken from the notes written by Marian Rejewski in 1967.

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As an example, we find the characteristics of a busy manoeuver day, when the following indicators were intercepted. This is a made up example taken from the notes written by Marian Rejewski in 1967.

The following table shows 64 intercepted characteristics from the same day. There are enough of them to find all three permutations AD, BE and CF.

A busy manoeuver day cont.

1. AUQ AMN
2. BNH CHL
3. BCT CGJ
4. CIK BZT
5. DDB VDV
6. EJP IPS
7. GPB ZSV
8. GPB ZSV
9. HNO THD
10. HNO THD
11. HXV TTI
12. IKG JKF
13. IKG JKF
14. IND JHU
15. JWF MIC
16. JWF MIC
17. KHB XJV
18. KHB XJV
19. LDR HDE
20. LDR HDE
21. MAW UXP
22. MAW UXP
23. NXD QTU
24. NXD QTU
25. NLU QFZ
26. OBU DLZ
27. PVJ FEG
28. QGA LYB
29. QGA LYB
30. RJL WPX
31. RJL WPX
32. RJL WPX
33. RJL WPX
34. RFC WQQ
35. SYX SCW
36. SYX SCW
37. SYX SCW
38. SYX SCW
39. SYX SCW
40. SJM SPO
41. SJM SPO
42. SJM SPO
43. SUG SMF
44. SUG SMF
45. TMN EBY
46. TMN EBY
47. TAA EXB
48. USE NWH
49. VII PZK
50. VII PZK
51. VQZ PVR
52. VQZ PVR
53. WTM RAO
54. WTM RAO
55. WTM RAO
56. WKI RKK
57. XRS GNM
58. XRS GNM
59. XOI GUK
60. XYW GCP
61. YPC OSQ
62. ZZY YRA
63. ZEF YOC
64. ZSJ YWG

back

back back

Characteristics of the manoeuver day

From the first indicator we find for example that

$$\mathtt{a}AD = \mathtt{a}, \quad \mathtt{u}BE = \mathtt{m}, \quad \mathtt{q}CF = \mathtt{n}.$$

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Similarly, the second indicator gives

$$\mathtt{b}AD = \mathtt{c}, \quad \mathtt{n}BE = \mathtt{h}, \quad \mathtt{h}CF = \mathtt{l}.$$

The following table lists the cycles of all three characteristics of the manoeuver day.

$$\begin{aligned} AD &= (\mathtt{a}), (\mathtt{s}), (\mathtt{bc}), (\mathtt{rw}), (\mathtt{dvpfkxgzyo}), (\mathtt{eijmunqlht}), \\ BE &= (\mathtt{axt}), (\mathtt{blfqveoum}), (\mathtt{cgy}), (\mathtt{d}), (\mathtt{hjpswizrn}), (\mathtt{k}), \\ CF &= (\mathtt{abviktjgfcqny}), (\mathtt{duzrehlxwpsmo}). \end{aligned}$$

More equations

By multiplying the corresponding pairs of the earlier found equations for the permutations A, B, C, D, E, F of the day we get the following system of equations:

$$\begin{aligned}
AD &= SHPNP^{-1}MLRL^{-1}M^{-1}PN^{-1}P^3NP^{-4}MLRL^{-1}\\
&\quad M^{-1}P^4N^{-1}P^{-4}H^{-1}S^{-1},\\
BE &= SHP^2NP^{-2}MLRL^{-1}M^{-1}P^2N^{-1}P^3NP^{-5}MLRL^{-1}\\
&\quad M^{-1}P^5N^{-1}P^{-5}H^{-1}S^{-1},\\
CF &= SHP^3NP^{-3}MLRL^{-1}M^{-1}P^3N^{-1}P^3NP^{-6}MLRL^{-1}\\
&\quad M^{-1}P^6N^{-1}P^{-6}H^{-1}S^{-1}.
\end{aligned}$$

back

How to simplify the system?

Now, not only the cyclic permutation was known, but also the three characteristics of the day on the left hand sides of the equations. But the system is certainly unsolvable for the unknown wirings of the three rotors and the reflector.

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The system can be formally simplified by substituting $Q = MLRL^{-1}M^{-1}$ into the three equations. The permutation Q is the wiring of a virtual reflector consisting of the reflector and the left and middle rotors that were fixed during the day. The substitution only leads to a slightly simplified system of equations on the next slide.

A slightly simplified system

$$\begin{aligned}
AD &= SHPNP^{-1}QPN^{-1}P^3NP^{-4}QP^4N^{-1}P^{-4}H^{-1}S^{-1},\\
BE &= SHP^2NP^{-2}QP^2N^{-1}P^3NP^{-5}QP^5N^{-1}P^{-5}H^{-1}S^{-1},\\
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This system is still certainly unsolvable for the unknown wirings N, Q even if all the other permutations are known. It has to be further simplified.

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Now comes the first of Marian Rejewski's ingenious ideas.

Blunders of German operators

When studying the tables of intercepted message keys he observed that the message keys were certainly not chosen randomly as the manual stated. There were too many repeated indicators.

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But if the operators did not choose the message keys randomly, what were the probable non random choices? Which stereotypes were probably used by the operators?

Marian Rejewski first proved the following simple theorem that helped him to understand the relationship between the two permutations A, D of a day and their composition, the characteristic AD of the same day. You will remember that each of the two permutations A and D contained only cycles of length 2.

One more theorem on permutations

Theorem. A permutation K on a set Z can be expressed as the composition of two permutations X, Y with all cycles of length two if and only if it contains an even number of cycles of each length.

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Theorem. A permutation K on a set Z can be expressed as the composition of two permutations X, Y with all cycles of length two if and only if it contains an even number of cycles of each length.

In order to understand why the theorem holds we first assume that $K = XY$ where both permutations X and Y have all cycles of length two.

Proof

We take a cycle (a_1, a_2) of the permutation X and investigate what are the lengths of the cycles of the product XY containing the elements a_1 and a_2. A picture will help.

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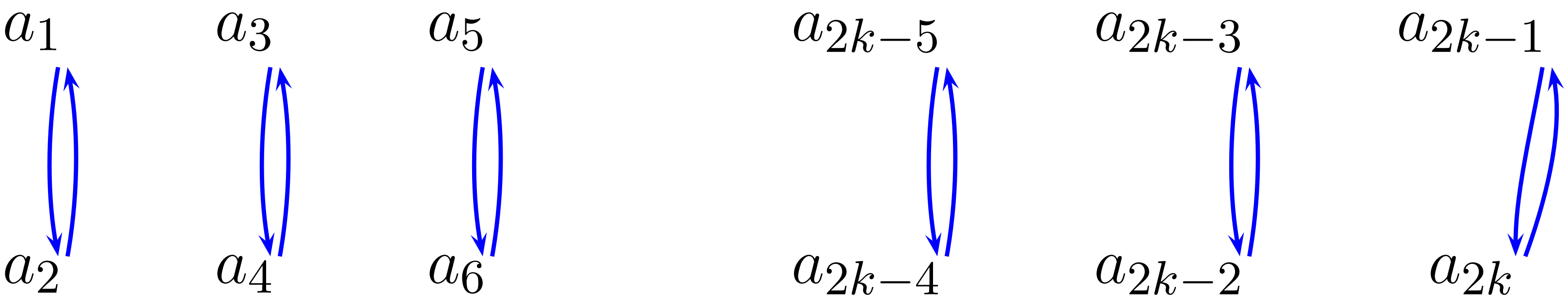

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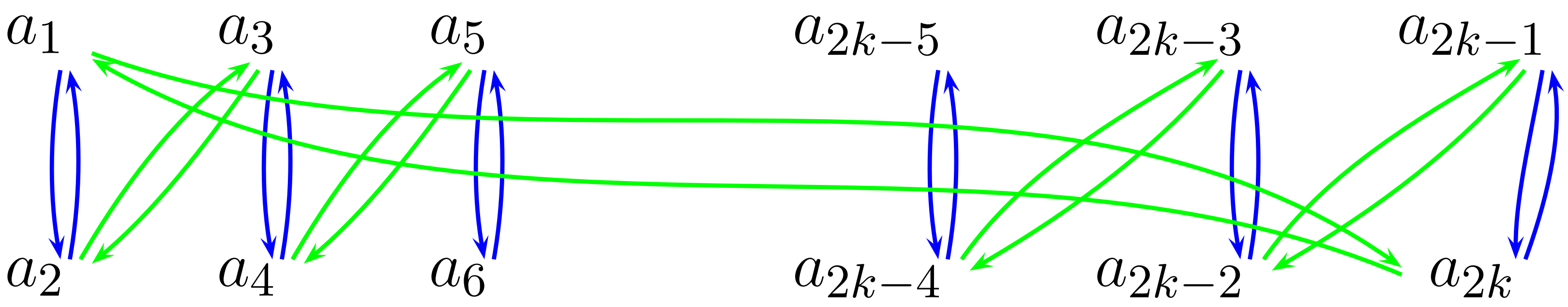
We take a cycle (a_1, a_2) of the permutation X and investigate what are the lengths of the cycles of the product XY containing the elements a_1 and a_2. A picture will help. We first draw the graph of X and then the graph of Y.



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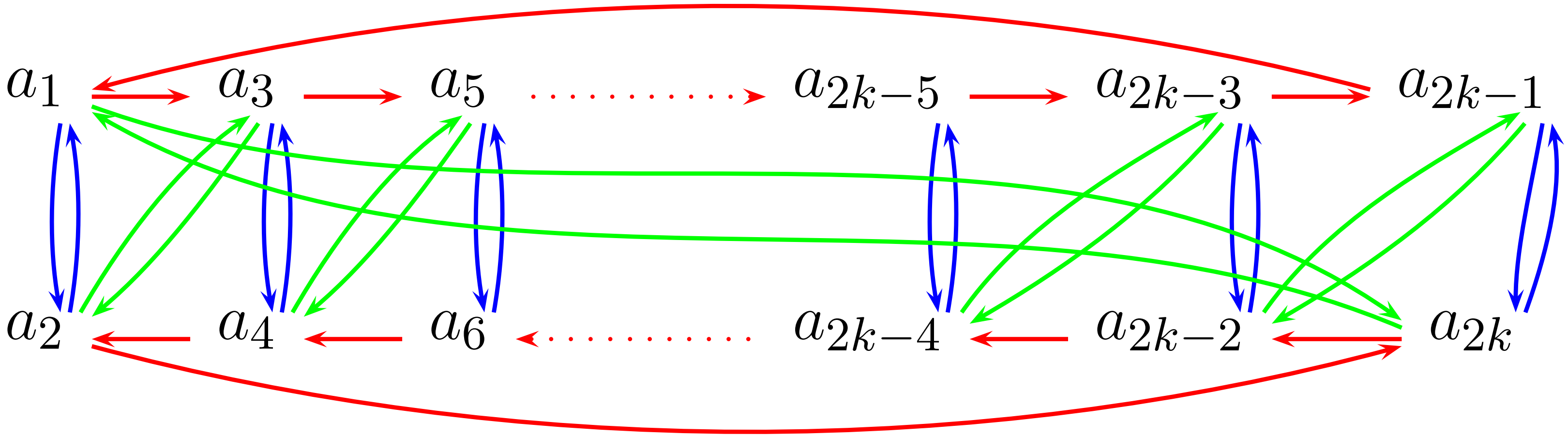
We take a cycle (a_1, a_2) of the permutation X and investigate what are the lengths of the cycles of the product XY containing the elements a_1 and a_2. A picture will help. We first draw the graph of X and then the graph of Y. Finally, we draw the graph of XY.



Proof cont.

We see that the elements a_1 and a_2 belong to two different cycles of the same length. They are $(a_1a_3\cdots a_{2k-3}a_{2k-1})$ and $(a_2a_{2k}a_{2k-2}\cdots a_4)$.

Proof cont.

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We choose two cycles of the same length, say k, and an arbitrary element a_1 in one of the cycles and an element a_2 in the other cycle. We may assume that $(a_1 a_2)$ is one of the cycles in X.

Proof cont.

We denote the two cycles $(a_1 a_3 \cdots a_{2k-3} a_{2k-1})$ and $(a_2 a_{2k} a_{2k-2} \cdots a_4)$, and draw them one under another and in the opposite direction.

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$$\begin{tikzcd}
a_1 \arrow[r] & a_3 \arrow[r] & a_5 \arrow[r, dotted] & a_{2k-5} \arrow[r] & a_{2k-3} \arrow[r] & a_{2k-1} \arrow[lllll, bend right] \\
a_2 \arrow[rrrrr, bend right] & a_4 \arrow[l] & a_6 \arrow[l] & a_{2k-4} \arrow[l, dotted] & a_{2k-2} \arrow[l] & a_{2k} \arrow[l]
\end{tikzcd}$$

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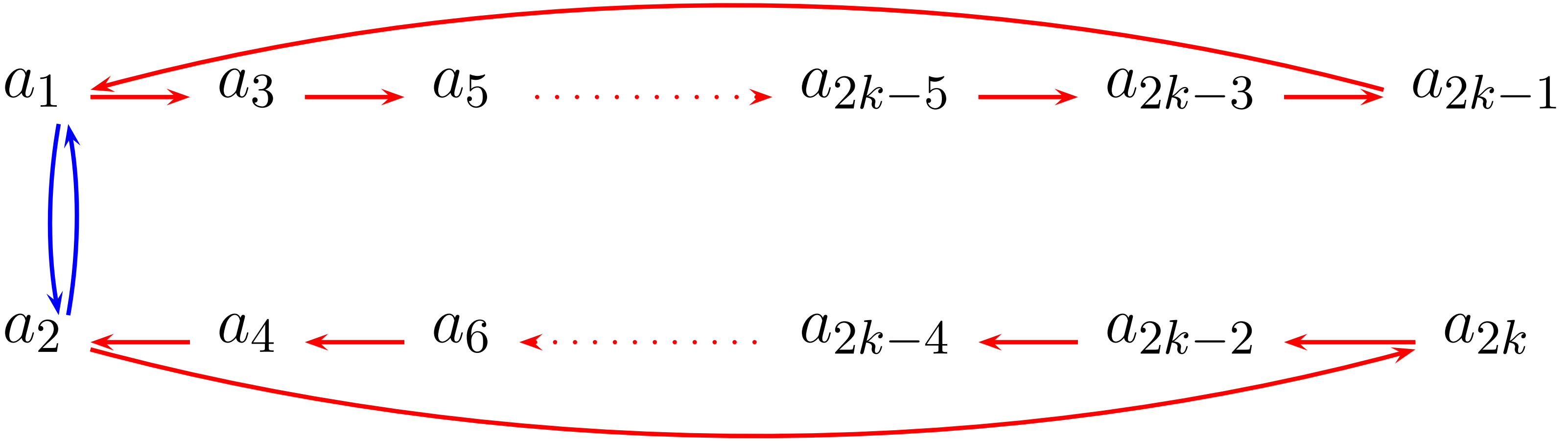

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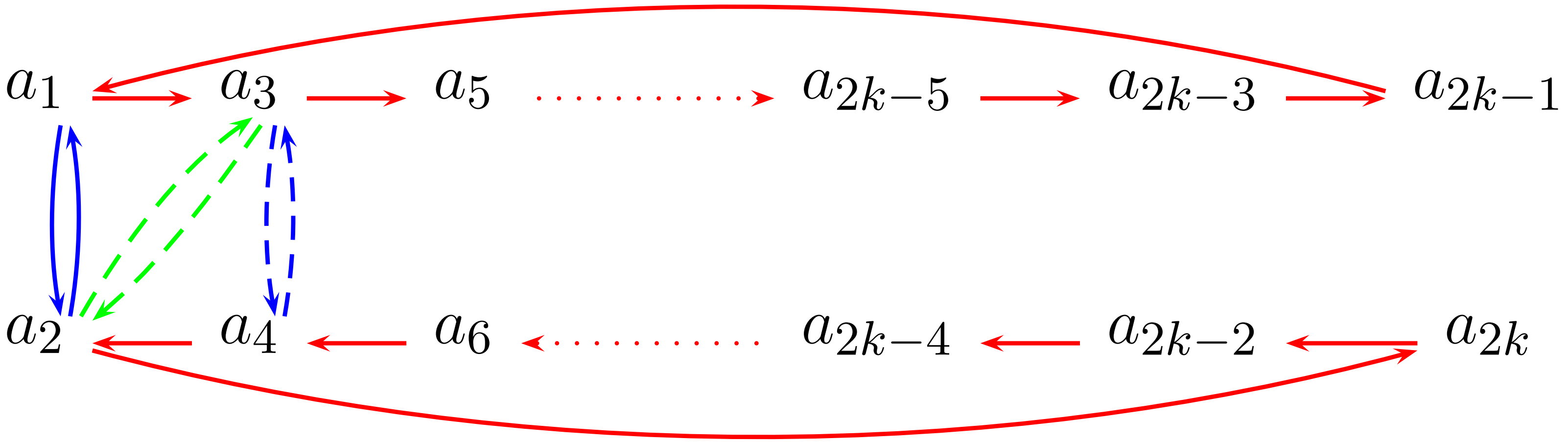
We denote the two cycles $(a_1 a_3 \cdots a_{2k-3} a_{2k-1})$ and $(a_2 a_{2k} a_{2k-2} \cdots a_4)$, and draw them one under another and in the opposite direction. Then we add the chosen cycle $(a_1 a_2)$ of permutation X. Since we want to have $K = XY$, the permutation Y must map the element a_2 to a_3. From the same reason, a_4 must be mapped to a_3 in X.



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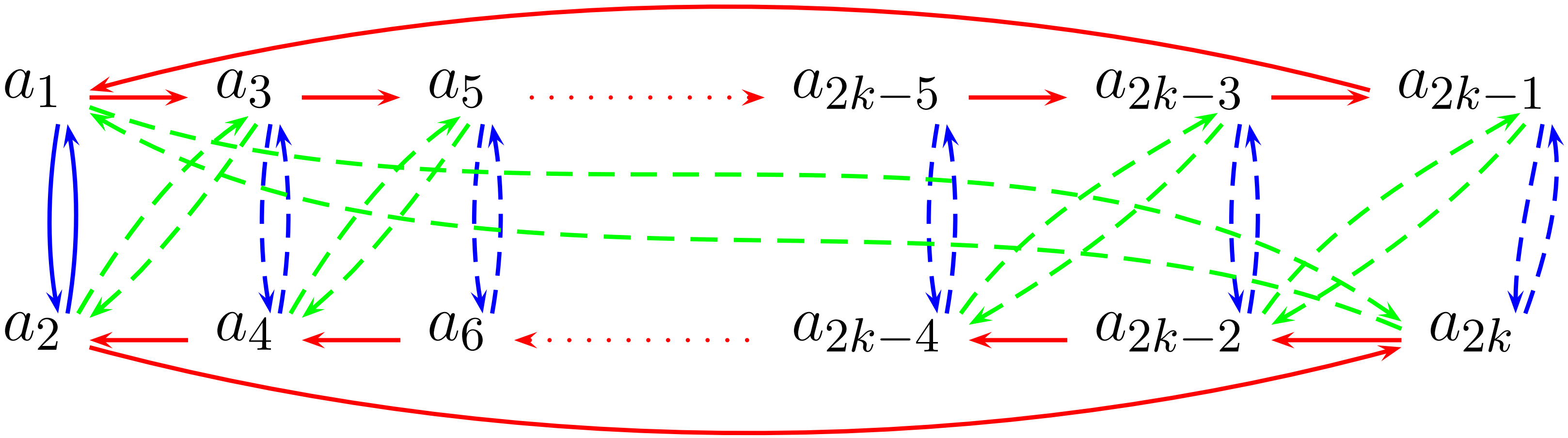

End of the proof

This procedure can be used for any pair of cycles of the same length. And since the permutation K has an even number of cycles of any given length, we may define in this way the permutations X and Y on all elements of the set Z.

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Note also that this procedure gives us a method how to find all decompositions of K as the product $K = XY$ of permutations with all cycles of length 2.

The number of possibilities

For example, the three characteristics of the Rejewski's example

$$\begin{aligned} AD &= (\mathtt{a}),(\mathtt{s}),(\mathtt{bc}),(\mathtt{rw}),(\mathtt{dvpfkxgzyo}),(\mathtt{eijmunqlht}),\\ BE &= (\mathtt{axt}),(\mathtt{blfqveoum}),(\mathtt{cgy}),(\mathtt{d}),(\mathtt{hjpswizrn}),(\mathtt{k}),\\ CF &= (\mathtt{abviktjgfcqny}),(\mathtt{duzrehlxwpsmo}) \end{aligned}$$

give 13 possibilities for the permutations C and F, 3×9 possibilities for the permutations B and E and 2×10 possibilities for the permutations A and D. All together $20 \times 27 \times 13 = 7020$ possibilities for the permutations of the day given these three characteristics of the day.
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Reducing the number of possibilities

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When he tried the possibility that the pattern `SYX SCW` was in fact the encryption of the message key AAA doubled, all of a sudden he was able to reconstruct the permutations A, B, C, D, E, F that gave very many stereotyped plain indicators.

A miracle

This assumption means that $\mathtt{a}A = \mathtt{s}$, $\mathtt{a}B = \mathtt{y}$ and $\mathtt{a}C = \mathtt{x}$.

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Since the elements a and y belong to different cycles of the same length 3 in the characteristic BE, this guess also determines the values of B and E on the six elements of the two cycles. The letters a and s form cycles of length 1 in AD, thus the cycle $(\mathtt{as})$ belongs to both A and D.

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With another two guesses of this form Marian Rejewski was eventually able to reconstruct all message keys used during the day. They are listed in the following table.

The message keys

AUQ AMN:	sss	IKG JKF:	ddd	QGA LYB:	xxx	VQZ PVR:	ert
BNH CHL:	rfv	IND JHU:	dfg	RJL WPX:	bbb	WTM RAO:	ccc
BCT CGJ:	rtz	JWF MIC:	ooo	RFC WQQ:	bnm	WKI RKK:	cde
CIK BZT:	wer	KHB XJV:	lll	SYX SCW:	aaa	XRS GNM:	qqq
DDB VDV:	ikl	LDR HDE:	kkk	SJM SPO:	abc	XOI GUK:	qwe
EJP IPS:	vbn	MAW UXP:	yyy	SUG SMF:	asd	XYW GCP:	qay
FBR KLE:	hjk	NXD QTU:	ggg	TMN EBY:	ppp	YPC OSQ:	mmm
GPB ZSV:	nml	NLU QFZ:	ghj	TAA EXB:	pyx	ZZY YRA:	uvw
HNO THD:	fff	OBU DLZ:	jjj	USE NWH:	zui	ZEF YOC:	uio
HXV TTI:	fgh	PVJ FEG:	tzu	VII PZK:	eee	ZSJ YWG:	uuu

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BCT CGJ:	rtz	JWF MIC:	ooo	RFC WQQ:	bnm	WKI RKK:	cde
CIK BZT:	wer	KHB XJV:	lll	SYX SCW:	aaa	XRS GNM:	qqq
DDB VDV:	ikl	LDR HDE:	kkk	SJM SPO:	abc	XOI GUK:	qwe
EJP IPS:	vbn	MAW UXP:	yyy	SUG SMF:	asd	XYW GCP:	qay
FBR KLE:	hjk	NXD QTU:	ggg	TMN EBY:	ppp	YPC OSQ:	mmm
GPB ZSV:	nml	NLU QFZ:	ghj	TAA EXB:	pyx	ZZY YRA:	uvw
HNO THD:	fff	OBU DLZ:	jjj	USE NWH:	zui	ZEF YOC:	uio
HXV TTI:	fgh	PVJ FEG:	tzu	VII PZK:	eee	ZSJ YWG:	uuu

With the exception of two message keys `abc` and `uvw` all the remaining ones are either triples of the same letters or triples of letters on neighbouring keys on the Enigma keyboard. And these two message keys are also far from being random.

A simplified system of equations

The psychological insight into the habits of German operators enabled Rejewski to return to the original system of equations.

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$$\begin{aligned}
A &= SHP^1NP^{-1}QP^1N^{-1}P^{-1}H^{-1}S^{-1}\\
B &= SHP^2NP^{-2}QP^2N^{-1}P^{-2}H^{-1}S^{-1}\\
C &= SHP^3NP^{-3}QP^3N^{-1}P^{-3}H^{-1}S^{-1}\\
D &= SHP^4NP^{-4}QP^4N^{-1}P^{-4}H^{-1}S^{-1}\\
E &= SHP^5NP^{-5}QP^5N^{-1}P^{-5}H^{-1}S^{-1}\\
F &= SHP^6NP^{-6}QP^6N^{-1}P^{-6}H^{-1}S^{-1}.
\end{aligned}$$

But now the permutations A, B, C, D, E, F were known.

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The daily keys were known!

In fact, Marian Rejewski reconstructed the message keys from a day in September 1932 and he moreover had the daily keys from this month. So he knew also the permutation S. He could move it to the left hand sides of the equations among the already known permutations. It gave him the following system.

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$$\begin{aligned}
S^{-1}AS &= HP^{1}NP^{-1}QP^{1}N^{-1}P^{-1}H^{-1} \\
S^{-1}BS &= HP^{2}NP^{-2}QP^{2}N^{-1}P^{-2}H^{-1} \\
S^{-1}CS &= HP^{3}NP^{-3}QP^{3}N^{-1}P^{-3}H^{-1} \\
S^{-1}DS &= HP^{4}NP^{-4}QP^{4}N^{-1}P^{-4}H^{-1} \\
S^{-1}ES &= HP^{5}NP^{-5}QP^{5}N^{-1}P^{-5}H^{-1} \\
S^{-1}FS &= HP^{6}NP^{-6}QP^{6}N^{-1}P^{-6}H^{-1}.
\end{aligned}$$

Germans like order

Now only the permutations H, N and Q were unknown. Rejewski first tried the same wiring between the plug board and the entry wheel that was used in the commercial Enigma. But it went nowhere.

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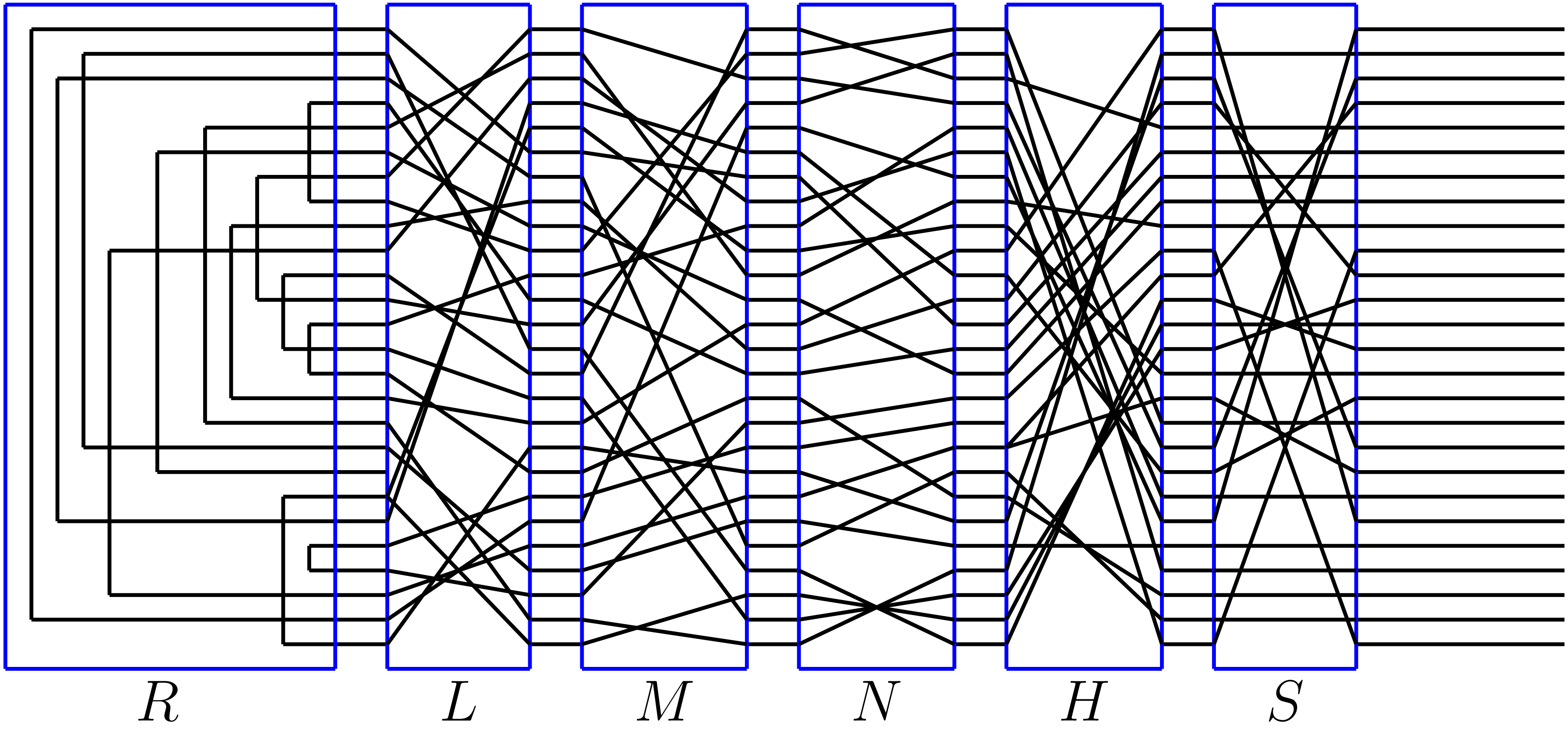
This unsuccessful attempt led him to the second ingenious insight into the psychology, this time of the constructors of the Enigma machine. Rejewski observed that the wiring between the plugboard and the entry wheel in the commercial Enigma machine was very regular. The plugs were connected to the entry wheel in the order of the keys on the keyboard. So he tried another regular wiring, this time in the order of the alphabet.

Real connections to the entry wheel



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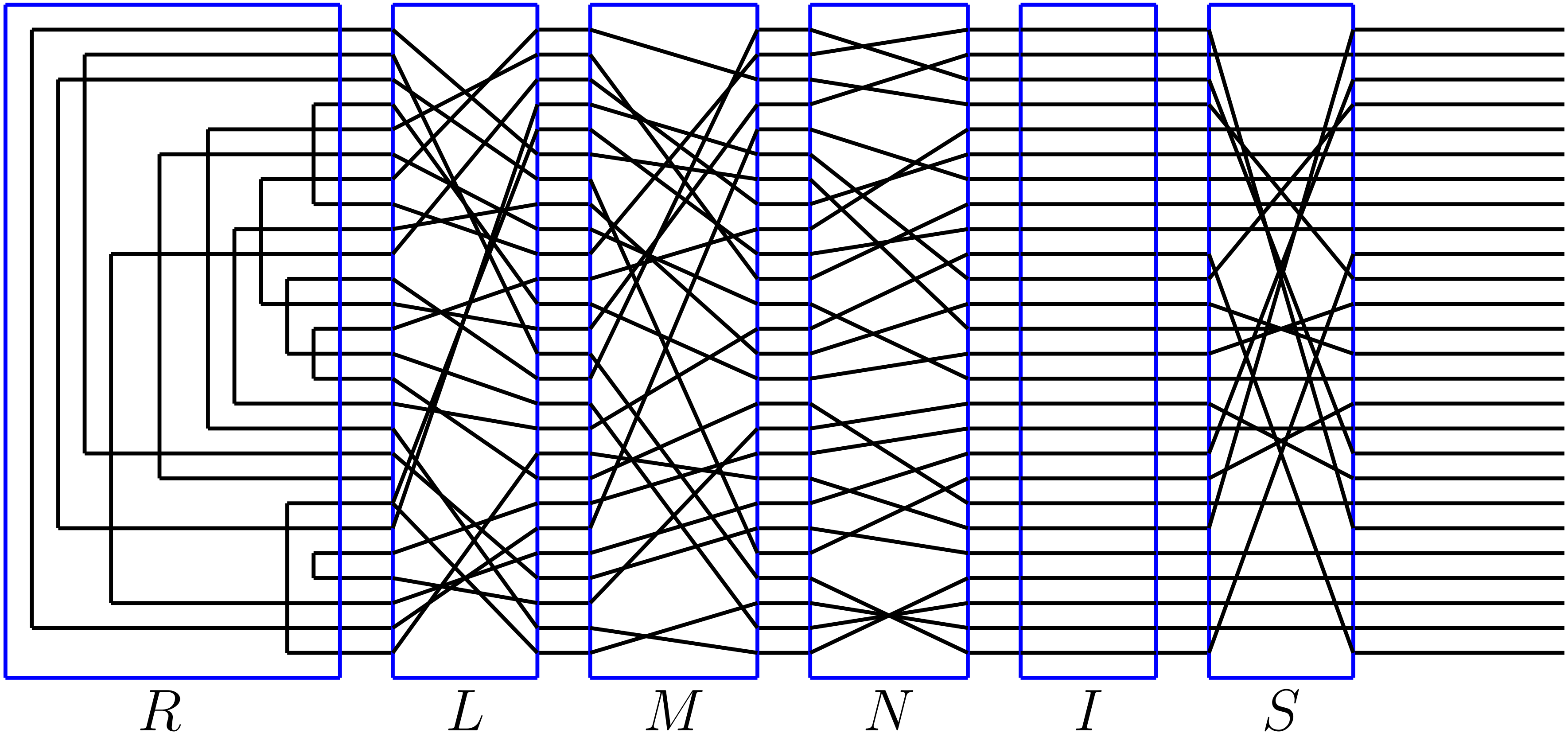

Thus the second try was $H = I$, the identity permutation.

Number of unknowns is getting smaller

Now only two permutations N and Q remained unknown. Rejewski rewrote the system of six equations in the following form.

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$$\begin{aligned}
T &= P^{-1}S^{-1}ASP^{1} = NP^{-1}QP^{1}N^{-1}, \\
U &= P^{-2}S^{-1}BSP^{2} = NP^{-2}QP^{2}N^{-1}, \\
W &= P^{-3}S^{-1}CSP^{3} = NP^{-3}QP^{3}N^{-1}, \\
X &= P^{-4}S^{-1}DSP^{4} = NP^{-4}QP^{4}N^{-1}, \\
Y &= P^{-5}S^{-1}ESP^{5} = NP^{-5}QP^{5}N^{-1}, \\
Z &= P^{-6}S^{-1}DSP^{6} = NP^{-6}QP^{6}N^{-1}.
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The moment of the truth

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$$\begin{aligned}
TU &= NP^{-1}(QP^{-1}QP)PN^{-1}, \\
UW &= NP^{-2}(QP^{-1}QP)P^2N^{-1}, \\
WX &= NP^{-3}(QP^{-1}QP)P^3N^{-1}, \\
XY &= NP^{-4}(QP^{-1}QP)P^4N^{-1}, \\
YZ &= NP^{-5}(QP^{-1}QP)P^5N^{-1}.
\end{aligned}$$

Only one unknown remained

From this system he eliminated the common expression $QP^{-1}QP$ and obtained the following system of four equations in one unknown N, or still better, in the unknown $V = NP^{-1}N^{-1}$.

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$$\begin{aligned}
UW &= NP^{-1}N^{-1}(TU)NPN^{-1} = V(TU)V^{-1}, \\
WX &= NP^{-1}N^{-1}(UW)NPN^{-1} = V(UW)V^{-1}, \\
XY &= NP^{-1}N^{-1}(WX)NPN^{-1} = V(WX)V^{-1}, \\
YZ &= NP^{-1}N^{-1}(XY)NPN^{-1} = V(XY)V^{-1}.
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\end{aligned}
$$

Moreover, all four equations have the familiar form

$$K = TLT^{-1}.$$

The solution

From the proof of the theorem that won the WWII we already know how to find all solutions V of each of the four equations. There must be a common solution V of the four equations that is a cyclic permutation, since it is conjugated to the the cyclic permutation P^{-1}. Hence he could also find all twenty six solutions for the permutation N.

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The wiring of one of the rotors thus became known. In those times the German army changed the order of rotors in daily keys every quarter of the year. Since September and October are in different quarters of the year, the same method led to the discovery of the wiring of another rotor. This was the rotor used as the right (fast) rotor in the other quarter of the year 1932 from which the daily keys were known.

Third rotor and reflector

Then it was possible to calculate the wiring of the remaining rotor and also of the reflector. Rejewski is very sketchy in this respect. Let us try to figure out how it could had been done.

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The first month the rotors were in the order L, M, N, from which the wiring N of the right rotor was calculated.

Third rotor and reflector cont.

Let us keep the notation also for the other month. The rotors were in a different order, certainly the right rotor had changed. This gives us four possibilities for the order of rotors in the other month:

- M, N, L, from which L can be calculated,

Third rotor and reflector cont.

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- N, L, M, from which M could be calculated,
- L, N, M, from which M could be calculated,

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- N, L, M, from which M could be calculated,
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- N, L, M, from which again M could be calculated.

In the first three of these cases the unknown rotor appears in at least one of the two month as the slowest left rotor.

Third rotor and reflector cont.

When calculating the wiring inside the fast right rotor, Rejewski used only intercepted messages from one day of the month. But he had at his disposal several dozens intercepted messages from each day of the month.

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So we may safely assume that in the tables of daily keys from the month he could find four days in which the positions of the left rotor formed an arithmetic sequence modulo 26. That means that the initial positions of the left rotor were

$$P^{x}LP^{-x}, P^{x+d}LP^{-x-d}, P^{x+2d}LP^{-x-2d}, P^{x+3d}LP^{-x-3d}$$

for some $x, d \in \{0, 1, 2, \ldots, 25\}$, preferably d odd.

Third rotor and reflector cont.

For each of these four days we take the equation expressing the first permutation of the day.

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So we get the system of equations

$$A_i = S_i N_i M_i P^{x+id} L P^{-x-id} R P^{x+id} L^{-1} P^{-x-id} M_i^{-1} N_i - 1 S_i^{-1}$$

for $i = 0, 1, 2, 3$ in the unknowns L, R. Here we have set $N_i = P^{y_i} N P^{-y_i}$ and $M_i = P^{z_i} M P^{-z_i}$ to simplify the notation. Moreover, the permutations S_i describe the plug board connections valid for the four chosen days.

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The rest of the calculation can be done in the way discovered by Rejewski.

Third rotor and reflector cont.

First we transfer as many of the known permutations as possible to the left hand side and obtain the equations

$$P^{-x-id}M_i^{-1}N_i^{-1}S_i^{-1}A_iS_iN_iM_iP^{x+id} = LP^{-x-id}RP^{x+id}L^{-1}$$

for i=0,1,2,3.

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Then we change the notation for the left hand sides to get

$$U_i = LP^{-x-id}RP^{x+id}L^{-1}.$$

Then we multiply pairs of subsequent equations and get for $i = 0, 1, 2$

$$U_iU_{i+1} = LP^{-x-id}RP^{-d}RP^dP^{x+id}L^{-1}.$$

Third rotor and reflector end

By eliminating the common expression $RP^{-d}RP^{d}$ from the three equations we get two equations for the unknown $LP^{d}L^{-1}$

$$U_iU_{i+1} = (LP^{d}L^{-1})U_{i+1}U_{i+2}(LP^{-d}L^{-1})$$

for $i = 0, 1$.

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for $i = 0, 1$.

From these two equations the unknown $LP^{d}L^{-1}$ can be calculated. Provided the joint solution of tese two equations is unique we subsequently get 26 possibilities for the unknown wiring L in case d is odd and 13×13 possibilities for L in case d is even.

Choice of the right wirings L, M, N

Then the wiring R inside the reflector can be easily and uniquely calculated from the equation

$$A = SP^1NP^{-1}MLRL^{-1}M^{-1}P^1N^{-1}P^{-1}S^{-1}$$

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As a result of these calculations Rejewski received at least $26 \times 26 \times 26 = 17576$ possibilities for the internal wirings of the Enigma machine. To choose the right one he was greatly helped by the example of a plain text and its real cipher version given in the operation manual.

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And finally, he found the position of the carry notches on the alphabet ring of each rotor.

How it was possible to calculate H

In his notes written in France probably in 1940 Rejewski mentions that it was also possible to calculate the permutation H describing the wiring between the plug board and the entry wheel. You will recall that Rejewski made a wild guess that H was the identity permutation and the guess proved to be true.

How it was possible to calculate H

In his notes written in France probably in 1940 Rejewski mentions that it was also possible to calculate the permutation H describing the wiring between the plug board and the entry wheel. You will recall that Rejewski made a wild guess that H was the identity permutation and the guess proved to be true.

To this end it was sufficient to find two days during the same month, in which the positions P^jNP^{-j} of the right rotor were the same. Since there were 26 possible positions and at least 30 days in the month, such two days always existed.

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We will denote the permutations defined by the virtual reflectors $M_1L_1RL_1^{-1}M_1^{-1}$ and $M_2L_2RL_2^{-1}M_2^{-1}$ in these two days by Q_1 and Q_2 respectively.

Calculating H cont.

The equations describing the permutations of the day in terms of the internal structure of the machine were for the first day

$$A_i = S_1 H P^{j+i} N P^{-j-i} Q_1 P^{j+i} N^{-1} P^{-j-i} H^{-1} S_1^{-1},$$

i=1,2,. . . ,6, and for the other day

$$B_i = S_2 H P^{j+i} N P^{-j-i} Q_2 P^{j+i} N^{-1} P^{-j-i} H^{-1} S_2^{-1},$$

i=1,2,. . . ,6.

Calculating H cont.

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$$B_i = S_2 H P^{j+i} N P^{-j-i} Q_2 P^{j+i} N^{-1} P^{-j-i} H^{-1} S_2^{-1},$$

i=1,2,. . . ,6.
Transferring the known permutations S_1 and S_2 to the left hand sides and multiplying the corresponding pairs of equations led to the equations ($i = 1, 2, \ldots, 6$)

$$S_1^{-1} A_i S_1 S_2^{-1} B_i S_2 = H P^{j+i} N P^{-j-i} Q_1 Q_2 P^{j+i} N^{-1} P^{-j-i} H^{-1}.$$

Calculating H end

By eliminating the common part Q_1Q_2 from all equations and by another use of the theorem that won WWII it was possible to find the permutations

$$HP^{j+i}(PN^{-1}P^{-1}N)P^{-j-i}H^{-1}$$

for $i = 1, 2, \ldots, 6$.

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Rejewski estimates that the lucky guess $H = I$ shortened the reconstruction of the Enigma machine by 3 months. Without this guess he would have to check not only $26^3 = 17576$ possibilities but $26^4 = 456976$ possibilities.

A replica of Enigma was built

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In July 1939, when it became clear that a new war in Europe was inevitable, the Polish Intelligence Service organized a meeting near Warsaw. There it passed the replicas and all other information to its French and British counterparts. This is how the first replica of the military Enigma machine got to Bletchley Park, the center of the British cryptanalysis of that time.

References

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Kozaczuk, Wladyslaw, *Enigma*, London: Arms and Armour, 1984,

Bauer, Friedrich L., *Decrypted Secrets, Methods and Maxims of Cryptology*, 2nd ed. Springer-Verlag, Berlin Heidelberg 2000.

http://www.spybooks.pl/en/enigma.html, a large collection of various notes written by Marian Rejewski, mainly in Polish.

Enigma simulators

Enigma simulators can be found e.g. at

www.xat.nl/enigma/

www.ugrad.cs.jhu.edu/ russell/classes/enigma/

frode.home.cern.ch/frode/crypto/simula/m3/