CVIČENÍ Z MATEMATIKY 2

Parciální derivace

Spočtěte vlastní parciální derivace následujících funkcí všude, kde existují:

1. $x^m y^n$

2. e^{xy}

3. $xy + yz + zx$

4. $\sqrt{x^2+y^2}$

5. $\sqrt[3]{x^3+y^3}$

6. $|x| \cdot |y|$

7. $\sqrt[3]{xy}$

8. $|y - \sin x|$

9. $|\sin y - \sin x|$

10. $\sqrt[3]{x+y^2}$

11. $f(x,y) = \begin{cases} \sqrt[3]{x^2+y} \cdot \ln(x^2+y^2), & [x,y] \neq [0,0], \\ 0, & [x,y] = [0,0], \end{cases}$

12. $f(x,y) = \begin{cases} e^{\frac{-1}{x^2+xy+y^2}}, & [x,y] \neq [0,0], \\ 0, & [x,y] = [0,0], \end{cases}$

13. $\left(\dfrac{x}{y}\right)^z$

14. $x^{\frac{y}{z}}$

15. x^{y^z}

Výsledky

1. $\frac{\partial f}{\partial x} = mx^{m-1}y^n$, $\frac{\partial f}{\partial y} = nx^m y^{n-1}$ pro $[x,y] \in \mathbb{R}^2$.

2. $\frac{\partial f}{\partial x} = ye^{xy}$, $\frac{\partial f}{\partial y} = xe^{xy}$ pro $[x,y] \in \mathbb{R}^2$.

3. $\frac{\partial f}{\partial x} = y + z$, $\frac{\partial f}{\partial y} = x + z$, $\frac{\partial f}{\partial z} = x + y$ pro $[x,y,z] \in \mathbb{R}^3$.

4. $\frac{\partial f}{\partial x}(x,y) = \frac{x}{\sqrt{x^2+y^2}}$, $\frac{\partial}{\partial y} f(x,y) = \frac{y}{\sqrt{x^2+y^2}}$, pokud $[x,y] \neq [0,0]$. $\frac{\partial}{\partial x} f(0,0)$ a $\frac{\partial f}{\partial y}(0,0)$ neexistují.

5. $\frac{\partial f}{\partial x}(x,y) = \frac{x^2}{\sqrt[3]{(x^3+y^3)^2}}$, $\frac{\partial}{\partial y} f(x,y) = \frac{y^2}{\sqrt[3]{(x^3+y^3)^2}}$, pokud $y \neq -x$. $\frac{\partial f}{\partial x}(0,0) = \frac{\partial f}{\partial y}(0,0) = 1$, $\frac{\partial f}{\partial x}(x,-x)$ a $\frac{\partial f}{\partial y}(x,-x)$ neexistují vlastní pro $x \neq 0$.

6. $\frac{\partial f}{\partial x}(x,y) = |y| \operatorname{sign} x$ pro $x \neq 0$. $\frac{\partial f}{\partial y}(x,y) = |x| \operatorname{sign} y$ pro $y \neq 0$. $\frac{\partial f}{\partial x}(0,0) = \frac{\partial f}{\partial y}(0,0) = 0$. $\frac{\partial f}{\partial x}(0,y)$ pro $y \neq 0$ a $\frac{\partial f}{\partial y}(x,0)$ pro $x \neq 0$ neexistují.

7. $\frac{\partial f}{\partial x}(x,y) = \frac{\sqrt[3]{y}}{3\sqrt[3]{x^2}}$ pro $x \neq 0$. $\frac{\partial f}{\partial y}(x,y) = \frac{\sqrt[3]{x}}{3\sqrt[3]{y^2}}$ pro $y \neq 0$. $\frac{\partial f}{\partial x}(0,0) = \frac{\partial f}{\partial y}(0,0) = 0$. $\frac{\partial f}{\partial x}(0,y)$ pro $y \neq 0$ a $\frac{\partial f}{\partial y}(x,0)$ pro $x \neq 0$ neexistují vlastní.

8. $\frac{\partial f}{\partial x}(x,y) = -\operatorname{sign}(y - \sin x) \cdot \cos x$, $\frac{\partial f}{\partial y}(x,y) = \operatorname{sign}(y - \sin x)$, pokud $y \neq \sin x$. $\frac{\partial f}{\partial y}(x, \sin x)$ neexistuje pro $x \in \mathbb{R}$. $\frac{\partial f}{\partial x}(\frac{\pi}{2} + k\pi, (-1)^k) = 0$ pro $k \in \mathbb{Z}$. $\frac{\partial}{\partial x} f(x, \sin x)$ neexistuje pro $x \neq \frac{\pi}{2} + k\pi$.

9. $\frac{\partial f}{\partial x}(x,y) = \cos x \operatorname{sign}(\sin x - \sin y)$, $\frac{\partial f}{\partial y}(x,y) = -\cos y \operatorname{sign}(\sin x - \sin y)$, pokud $\sin x \neq \sin y$. $\frac{\partial f}{\partial x}(\frac{\pi}{2} + k\pi, \frac{\pi}{2} + l\pi) = \frac{\partial}{\partial y} f(\frac{\pi}{2} + k\pi, \frac{\pi}{2} + l\pi) = 0$. V ostatních bodech parciální derivace neexistují.

10. $\frac{\partial f}{\partial x}(x,y) = \frac{1}{3\sqrt[3]{x+y^2}}$, $\frac{\partial}{\partial y} f(x,y) = \frac{2y}{3\sqrt[3]{x+y^2}}$, pokud $x \neq -y^2$, $\frac{\partial f}{\partial x}(-x^2, x)$ a $\frac{\partial f}{\partial y}(-x^2, x)$ neexistují pro $x \in \mathbb{R}$.

11. V bodech $\mathbb{R}^2 \setminus \{[x, -x^2], x \in \mathbb{R}\}$ je $\frac{\partial f}{\partial x} = \frac{2x \ln(x^2+y^2)}{3\sqrt[3]{x^2+y}^2} + \frac{2x\sqrt[3]{x^2+y}}{x^2+y^2}$ a $\frac{\partial f}{\partial y} = \frac{\ln(x^2+y^2)}{3\sqrt[3]{x^2+y}^2} + \frac{2y\sqrt[3]{x^2+y}}{x^2+y^2}$. V $[x, -x^2]$ neexistují vlastní parciální derivace pokud $x^2 + x^4 \neq 1$, pokud $x^2 + x^4 = 1$, jsou obě parciální derivace nulové.

12. $\frac{\partial f}{\partial x} = e^{\frac{-1}{x^2+xy+y^2}} \cdot \frac{2x+y}{(x^2+xy+y^2)^2}$, $\frac{\partial f}{\partial y} = e^{\frac{-1}{x^2+xy+y^2}} \cdot \frac{x+2y}{(x^2+xy+y^2)^2}$ pro $[x, y] \neq [0, 0]$; v bodě $[0, 0]$ jsou obě parciální derivace nulové.

13. Pokud $x, y > 0$ nebo $x, y < 0$, pak $\frac{\partial f}{\partial x} = \frac{z}{y} \cdot \left(\frac{x}{y}\right)^{z-1}$; $\frac{\partial f}{\partial y} = -\frac{zx}{y^2} \cdot \left(\frac{x}{y}\right)^{z-1}$; $\frac{\partial f}{\partial z} = \left(\frac{x}{y}\right)^{z} \cdot \log \frac{x}{y}$.

14. Pokud $x > 0$ a $y \neq 0$, pak $\frac{\partial f}{\partial x} = \frac{y}{z} \cdot x^{\frac{y}{z}-1}$; $\frac{\partial f}{\partial y} = x^{\frac{y}{z}} \cdot \log x \cdot \frac{1}{z}$; $\frac{\partial f}{\partial z} = -x^{\frac{y}{z}} \cdot \log x \cdot \frac{y}{z^2}$.

15. Pokud $x, y > 0$, pak $\frac{\partial f}{\partial x} = y^z \cdot x^{y^z-1}$; $\frac{\partial f}{\partial y} = x^{y^z} \cdot \log x \cdot zy^{z-1}$; $\frac{\partial f}{\partial z} = x^{y^z} \cdot \log x \cdot y^z \cdot \log y$.