CVIČENÍ Z MATEMATICKÉ ANALÝZY 2

KONVERGENCE NEWTONOVA INTEGRÁLU

Vyšetřete konvergenci Newtonova integrálu (v závislosti na parametru $\alpha, \beta \in \mathbb{R}$).

(1) $\int_{-1}^{1} \frac{1}{\sqrt{1-x^4}} \, dx$

(2) $\int_{0}^{1} \frac{x^2}{\sqrt[3]{(1-x^2)^5}} \, dx$

(3) $\int_{0}^{\infty} \frac{1-\cos x}{x^{\frac{5}{2}}} \, dx$

(4) $\int_{0}^{1} \frac{1}{e^{\sqrt{x}}-1} \, dx$

(5) $\int_{0}^{1} \frac{\operatorname{tg} x}{\sqrt{x^3}} \, dx$

(6) $\int_{0}^{\infty} x^{-3/4} e^{\sqrt{x}} \, dx$

(7) $\int_{0}^{\infty} \frac{1}{\sqrt{x} \log(1+e^x)}$

(8) $\int_{0}^{\frac{\pi}{2}} \frac{\log \sin x}{\sqrt{x}} \, dx$

(9) $\int_{-\infty}^{\infty} e^{-x^2} \, dx$

(10) $\int_{0}^{\frac{\pi}{2}} \sin\left(\frac{1}{\sin x}\right) \, dx$

(11) $\int_{0}^{\infty} \frac{x^\alpha}{1+x^\beta} \, dx$

(12) $\int_{0}^{1} x^{\alpha-1}(1-x)^{\beta-1} \, dx$

(13) $\int_{0}^{\infty} \frac{1-\cos x}{x^\alpha} \, dx$

(14) $\int_{1}^{e} \frac{(\log x)^\alpha}{x} \, dx$

(15) $\int_{0}^{1/e} \frac{|\log x|^\alpha}{x} \, dx$

(16) $\int_{0}^{\frac{\pi}{2}} (\operatorname{tg} x)^\alpha \, dx$

(17) $\int_{0}^{\infty} x^\alpha \operatorname{arctg}^\beta x \, dx$

(18) $\int_{0}^{\infty} \frac{1}{x^\alpha+x^\beta} \, dx$

(19) $\int_{0}^{1} \frac{\arccos x}{\log^\alpha \frac{1}{x}} \, dx$

(20) $\int_{0}^{\infty} (2\pi - \operatorname{arctg} x)^\alpha \, dx$

VÝSLEDKY

(1) K

(2) D

(3) K

(4) K

(5) K

(6) D

(7) K

(8) K

(9) K

(10) K

(11) K $\Leftrightarrow 0 < \alpha + 1 < \beta$

(12) K $\Leftrightarrow \alpha, \beta > 0$

(13) K $\Leftrightarrow 1 < \alpha < 3$

(14) K $\Leftrightarrow \alpha > -1$

(15) K $\Leftrightarrow \alpha < -1$

(16) K $\Leftrightarrow \alpha \in (-1, 1)$

(17) K $\Leftrightarrow \alpha < -1 < \alpha + \beta$

(18) K $\Leftrightarrow \min\{\alpha, \beta\} < 1 < \max\{\alpha, \beta\}$

(19) K $\Leftrightarrow \alpha < \frac{3}{2}$

(20) K $\Leftrightarrow \alpha > 1$