

CVIČENÍ Z MATEMATICKÉ ANALÝZY 2

KONVERGENCE ŘAD

Vyšetřete konvergenci řady (v závislosti na parametru $a \in \mathbb{R}$, je-li uveden):

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| <p>(1) $\sum_{n=1}^{\infty} \frac{n^2 + 1}{n^3 + 4}$</p> <p>(2) $\sum_{n=1}^{\infty} \frac{n^5 + 3n^3 + n + 1}{n^4 + 2n^6 + n^7 + 6}$</p> <p>(3) $\sum_{n=1}^{\infty} n^a (\sqrt{n+1} - \sqrt{n})$</p> <p>(4) $\sum_{n=2}^{\infty} \frac{1}{\sqrt[n]{\log n}}$</p> <p>(5) $\sum_{n=1}^{\infty} \frac{n\sqrt{n} + 2n}{n^2 + 2n^3}$</p> <p>(6) $\sum_{n=1}^{\infty} \frac{1}{\sqrt{2n+1}\sqrt{2n+3}}$</p> <p>(7) $\sum_{n=1}^{\infty} (\sqrt{n^3+1} - \sqrt{n^3-1})$</p> <p>(8) $\sum_{n=1}^{\infty} n^3 \left(1 - \cos \frac{1}{n}\right)$</p> <p>(9) $\sum_{n=1}^{\infty} \log \left(1 + \frac{1}{n^2}\right)$</p> <p>(10) $\sum_{n=1}^{\infty} \frac{\arctan n}{n}$</p> <p>(11) $\sum_{n=1}^{\infty} \sin \frac{1}{n} \log \frac{n+1}{n}$</p> <p>(12) $\sum_{n=1}^{\infty} \frac{n}{n^2+1} \cos \frac{1}{n}$</p> <p>(13) $\sum_{n=1}^{\infty} \frac{\sin \left(\frac{1}{\sqrt{n}} - \frac{1}{\sqrt{n+1}}\right)}{\sqrt[3]{n^2+1} - \sqrt[3]{n^2}}$</p> <p>(14) $\sum_{n=1}^{\infty} \arctan \frac{2n}{1+n^2}$</p> <p>(15) $\sum_{n=1}^{\infty} \frac{1}{n} \arccos \frac{1}{n}$</p> <p>(16) $\sum_{n=1}^{\infty} \frac{n^5}{5^n}$</p> <p>(17) $\sum_{n=1}^{\infty} \frac{2^n}{n!}$</p> | <p>(18) $\sum_{n=2}^{\infty} \frac{1}{n \log^2 n}$</p> <p>(19) $\sum_{n=1}^{\infty} \left(\frac{1 + \cos n}{2 + \cos n}\right)^n$</p> <p>(20) $\sum_{n=1}^{\infty} \frac{2^n}{3^n}$</p> <p>(21) $\sum_{n=1}^{\infty} \frac{1}{\left(2 + \frac{1}{n}\right)^n}$</p> <p>(22) $\sum_{n=1}^{\infty} \frac{n!}{2n^2}$</p> <p>(23) $\sum_{n=1}^{\infty} \frac{n^5}{2^n + 3^n}$</p> <p>(24) $\sum_{n=1}^{\infty} \frac{n^{n+\frac{1}{n}}}{\left(n + \frac{1}{n}\right)^n}$</p> <p>(25) $\sum_{n=1}^{\infty} \left(\frac{2 + (-1)^n}{7}\right)^n$</p> <p>(26) $\sum_{n=2}^{\infty} \left(\frac{n-1}{n+1}\right)^{n(n-1)}$</p> <p>(27) $\sum_{n=1}^{\infty} \binom{2n}{n} \frac{1}{5^n}$</p> <p>(28) $\sum_{n=1}^{\infty} \frac{3}{2^n - 2n}$</p> <p>(29) $\sum_{n=1}^{\infty} \frac{2^n + (-1)^n n}{3^n + (-1)^n n}$</p> <p>(30) $\sum_{n=1}^{\infty} \left(n^{(n^2+1)^{-1}} - 1\right)$</p> <p>(31) $\sum_{n=1}^{\infty} \frac{\sqrt[3]{n+2} - \sqrt[3]{n+1}}{n^a}$</p> <p>(32) $\sum_{n=1}^{\infty} \frac{\sqrt[3]{n^2+5} - \sqrt[3]{n^2+1}}{\sqrt[4]{n}}$</p> <p>(33) $\sum_{n=1}^{\infty} \frac{3^n + 4^n}{4^n + 5^n}$</p> <p>(34) $\sum_{n=1}^{\infty} \frac{a^n}{n}$</p> <p>(35) $\sum_{n=1}^{\infty} n^3 a^n$</p> |
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$$(36) \sum_{n=0}^{\infty} \frac{2^n}{n!} a^n$$

$$(37) \sum_{n=1}^{\infty} \frac{n^3}{3^n} a^n$$

$$(38) \sum_{n=1}^{\infty} \frac{a^n}{n^2}$$

$$(39) \sum_{n=1}^{\infty} (-1)^n \frac{2n^2 + 3n + 4}{2n^2 + 5}$$

$$(40) \sum_{n=1}^{\infty} (-1)^n \frac{2n^2 + 3n + 4}{(2n^2 + 5)^2}$$

$$(41) \sum_{n=0}^{\infty} \frac{(-1)^n a^{2n+1}}{2n+1}$$

$$(42) \sum_{n=1}^{\infty} (-1)^n (\sqrt[n]{3} - 1)$$

$$(43) \sum_{n=2}^{\infty} (-1)^n \frac{1}{\log n}$$

$$(44) \sum_{n=1}^{\infty} (-1)^n \frac{n}{n^2 + 2}$$

$$(45) \sum_{n=1}^{\infty} (-1)^n \left(\frac{2n+10}{3n+1} \right)^n$$

$$(46) \sum_{n=1}^{\infty} (-1)^{n+1} \frac{a^n}{n}$$

$$(47) \sum_{n=1}^{\infty} \cos(n^2\pi) (\sqrt{n+9} - \sqrt{n})$$

$$(48) \sum_{n=1}^{\infty} \frac{(-1)^n}{2n + (-1)^n}$$

VÝSLEDKY

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| (1) D | (18) K | (35) $K \Leftrightarrow a \in (-1, 1)$ |
| (2) K | (19) K | (36) $K \Leftrightarrow a \in \mathbb{R}$ |
| (3) $K \Leftrightarrow a < -\frac{1}{2}$ | (20) K | (37) $K \Leftrightarrow a \in (-3, 3)$ |
| (4) D | (21) K | (38) $K \Leftrightarrow a \in [-1, 1]$ |
| (5) K | (22) K | (39) D |
| (6) D | (23) K | (40) K (absolutně) |
| (7) K | (24) D | (41) $K \Leftrightarrow a \in [-1, 1]$ |
| (8) D | (25) K | (42) K (neabsolutně) |
| (9) K | (26) K | (43) K (neabsolutně) |
| (10) D | (27) K | (44) K (neabsolutně) |
| (11) K | (28) K | (45) K (absolutně) |
| (12) D | (29) K | (46) $AK \Leftrightarrow a \in (-1, 1),$
$K \Leftrightarrow a \in (-1, 1]$ |
| (13) D | (30) K | (47) K (neabsolutně) |
| (14) D | (31) $K \Leftrightarrow a > \frac{1}{3}$ | (48) K (neabsolutně) |
| (15) D | (32) K | |
| (16) K | (33) K | |
| (17) K | (34) $K \Leftrightarrow a \in [-1, 1)$ | |