

Výsledky příkladů

Cvičení 8

1. (a) $c = 24$
(b) $f_X(z) = 12z(1-z)^2 \mathbb{I}\{z \in (0, 1)\} = f_Y(z)$, nejsou nezávislé
(c) $P(X > Y) = 1/2$
(d) $\text{Cov}(X, Y) = -\frac{2}{75}$, $\text{Var}(X) = \frac{1}{25} = \text{Var}(Y)$
(e) $E(\frac{1}{XY}) = 12$
(f) $f_{Y|X}(y|x) = \frac{2y}{(1-x)^2} \mathbb{I}\{y \in (0, 1-x)\}$ pro $x \in (0, 1)$. Pro $x \notin (0, 1)$ není podmíněná hustota definována.
(g) $E[Y | X] = \frac{2}{3}(1-X)$
(h) $\int_0^1 \frac{2}{3}(1-x)f_X(x)dx = \frac{2}{5}$
(i) $\text{Var}[Y | X] = \frac{1}{18}(1-X)^2$
2. (a) $c = 6/7$
(b) $E[Y | X = x] = \frac{2}{3}x$ pro $x \in (1, 2)$, jinak nedefinováno.
 $E[Y | X] = \frac{2}{3}X$
(c) $E(YX^3) = \frac{254}{49}$, $E[YX^3 | X] = \frac{2}{3}X^4$
3. $\text{Var}(\mathbf{Z}) = \begin{pmatrix} \frac{3}{80} & \frac{1}{80} \\ \frac{1}{80} & \frac{3}{80} \end{pmatrix}$
 $P(X + Y < 1) = \frac{1}{2}$
4. $E[Y | X] = X$
 $\text{Var}[Y | X] = X^2$
 $\text{Var}(Y) = \frac{29}{12}$
5. (a) $F_{X_*}(x) = 1 - (1 - F_X(x))^n$
(b) $F_{X^*}(x) = F_X(x)^n$
(c) $f_{X_*}(x) = n(1 - F_X(x))^{n-1}f_X(x)$, $f_{X^*}(x) = n(F_X(x))^{n-1}f_X(x)$
(d) $E(X_*) = \lambda n$