

Life Insurance 2 - HW3

Part 1 (5 points)

Let $J = 1$ represent death by accidental means and $J = 2$ death by other means. Assume that $\delta = 0.05$, $\mu_{1,x+t} = 0.005$ for $t \geq 0$, where $\mu_{1,x+t}$ is the force of decrement for death by accidental means and $\mu_{2,x+t} = 0.02$ for $t \geq 0$.

A 20-year term insurance policy, payable at the moment of death, is issued to a life age x providing a benefit of 2 if death is by accidental means and providing a benefit of 1 for other deaths. Find the expectation and variance of the present value of benefits random variable.

Part 2 (5 points)

Assume the following compound model with one decrement:

x	$q'_{1,x}$	$q'_{2,x}$	$q'_{3,x}$
65	0.020	0.02	0.04
66	0.025	0.02	0.06
67	0.030	0.02	0.08
68	0.035	0.02	0.10
69	0.040	0.02	0.12

1. Under the assumption of linearity for each cause of decrement compute $q_{1,x}$, $q_{2,x}$ and $q_{3,x}$.
2. Now, assume that columns $q'_{1,x}$, $q'_{2,x}$ and $q_{3,x}$ are known. Derive $q'_{3,x}$.

Hint for task 2.: Realize that $q_{j,x} = \int_0^1 {}_t p_x \cdot \mu_{j,x+t} dt$. Use the assumption of linearity to get rid of $\mu_{j,x+t}$.

Part 3 (4 points)

Use the life tables that you created in the winter semester and the assumed $i = 0.8\%$ to calculate the joint life annuity $a_{x:y}$ and the last survivor annuity $a_{\overline{x:y}}$ for independent lives age $x = 65$ (male) and $y = 60$ (female). You can use life tables created by you or these ones: [link to life tables](#).

Part 4 (6 points)

Compute and plot the gross premium reserves for

1. the endowment insurance until 70 years with the net annual premium collected during the whole period with SI=1,000,000,
2. the life annuity in advance deferred until the age 70 years with the net annual premium collected during the deferment period with SI=120,000.

Consider the input ages $x = 25, 30$ and 35 years. Use the unisex life tables (TIR=0.8%) and the corresponding commutation functions. The expenses values are

- $\alpha = 5\%$
- $\beta = 0.8\%$
- $\gamma = 2\%$
- $\delta = 0.5\%$

Add a few words about the obtained results and include also the general formulas for the premium and reserves in your solution.

Send a PDF file *surname_name_HW3.pdf* to vejmel@karlin.mff.cuni.cz until May 10, 2022.