

# Mathematics of Life Insurance 1 - HW1

## Part 1 (4 points)

Consider a mortgage for 54,000\$ that should be paid by monthly payments (paid at the end of each month) of the same amount in 30 years with a nominal interest rate of 4%.

1. Calculate the monthly payment.
2. For each payment compute what part of the current debt is paid by the payment and plot these values.

## Part 2 (8 points)

Let

$$P(T_0 > t) = \exp \left\{ - \left( Kt + \frac{1}{2}Lt^2 + \frac{M}{\log N}N^t - \frac{M}{\log N} \right) \right\},$$

where  $K, L, M$  and  $N$  are all positive.

1. Derive a formula for  $P(T_x > t)$ .
2. Derive a formula for  $\mu_x$ .
3. Consider

$$K = 0.0001, L = 0.000007, M = 0.00035, N = 1.08.$$

- (a) Calculate  ${}_t p_{20}$  for  $t = 1, 10, 50, 90$ .
- (b) Calculate  ${}_t q_{30}$  for  $t = 1, 10, 50$ .
- (c) Calculate  ${}_{t|15}q_{35}$  for  $t = 1, 10, 50$ .
- (d) Calculate  $\ddot{e}_x$  for  $x = 65, 66, 67, 68, 69, 70$ .

## Part 3 (5 points)

Compute commutation functions ( $C_x, D_x, M_x, N_x, R_x, S_x$ ) for **Men, Women** and **Unisex** using Czech life tables for **2022**. Use TIR =  $i = 2\%$ . For preparing Unisex tables use  $l_{Unisex} = 0.5 \cdot l_{Men} + 0.5 \cdot l_{Women}$ .

When the commutation functions are calculated, plot the net single premiums for the following capital life insurances for ages  $x = 20, \dots, 60$  and all three life tables.

1. Pure endowment for  $n = 65 - x$  years,
2. Term insurance until 65 years, i.e., for  $n = 65 - x$  years.

For each insurance give one graph with three lines with respect to used life tables and add a short comment explaining the behaviour of the net single premiums.

## Part 4 (4 points)

Consider Unisex life tables for the Czech Republic for year 2022.

Calculate  ${}_{0.2}q_{52.4}$

1. under the linearity assumption,
2. under the assumption of constant force of mortality.

## Part 5 (5 points)

Consider term insurance for 20 years issued to a life aged  $x$ . The benefit is paid at the end of the year of death and is of amount 1 if death occurs during the first year,  $1 + g$  during the second year,  $(1 + g)^2$  during the third year, and so on.

Plot the net single premium considering Unisex life tables and  $g = 0.5\%$  for ages  $x = 20, \dots, 80$ .

## Part 6 (4 points)

Suppose that a life aged 30 arranged an insurance with the following parameters. If death occurs in the first 20 years, 10,000 is paid. Otherwise, 20,000 is paid. Moreover, it was arranged with the insurance company that a premium refund will be paid in the case of death during the first 5 years. This refund will be 25% of the **net single premium**. Assume that it holds:  $l_x = 100 - x$ ,  $0 \leq x \leq 100$  and  $i = 5\%$ .

Calculate the net single premium of this insurance.

You do not have to create Word/LaTeX/... form of your solution. A handwritten (but legible) solution with added graphs is completely fine.

**Send your solution as a PDF file *surname\_name\_HW1.pdf* to [vejmelp@karlin.mff.cuni.cz](mailto:vejmelp@karlin.mff.cuni.cz) until November 24, 2023.**