Mathematics of Life Insurance 1 - HW2

Part 1 (6 points)

Assume that T_x (remaining lifetime of a specific life aged x) has a probability density function given by

$$f_{T_x}(t) = \frac{2}{10 \cdot \sqrt{2\pi}} \cdot e^{-\frac{t^2}{200}} \qquad t > 0.$$

Furthermore, suppose $\delta = 0.05$.

Calculate $\overline{A}_x$ and the median of Z, where $Z = v^T$.

Part 2 (6 points)

Similarly as in the previous homework plot the net single premiums for the following life annuities for ages (x from 20 to 60 per 1 year) and all three life tables (men, women, unisex) that you have constructed.

1. Life annuity in advance for $n = 65 - x$ years,
2. $m = 65 - x$ year deferred life annuity in advance (payments start at age 65).

In addition to that, for the deferred life annuity in advance (2.) plot also the annual net premium, which is paid during the deferment period.

For each insurance give one graph with three lines with respect to used life tables and add a short comment explaining the behaviour of the net single premiums/annual net premium.

Part 3 (6 points)

Assume a whole life insurance which was arranged by a person of age x. Let the following be known: annual net premium $P = 0.048$ paid until death, $A_x = 0.4$, ${}^2A_x = 0.2$ and $d = 0.06$. As usual, L is the insurer's loss at issue of this policy.

1. Calculate E[L] and Var[L].
2. Consider a portfolio of 100 policies of this type, where

Sum Insured	Number of Policies
1	80
4	20

   Assume the losses are independent and use a normal approximation to calculate the probability that the present value of **gains** (gain is considered as -loss) for the portfolio will exceed 20.

Remark: In this case the principle of equivalence does not hold.

Part 4 (3 points)

Assume an m-year deferred temporary standard increasing life annuity in arrear for n years, where the premium is paid $m_1 < m$ years. Write down the form of L.

Part 5 (9 points)

Consider a whole life insurance issued to a life aged 25 that pays a unit at the end of the year of death. It was arranged that premiums will be paid annually to the age of 65. However, this premium does not have a constant value.

- During the first 10 years, premium P_1, corresponding to the annual premium that should be paid in case of paying constant premium for this whole life insurance till death, is paid.
- The following 30 years, an increased premium P_2 is paid.

For the following tasks use the Unisex life tables.

1. Calculate the annual premium P_2 payable at ages 35 through 64.
2. Calculate the tenth-year reserve.
3. After 10 years, the policyholder has the option not to pay the increased premium P_2, but he can decide to continue with premium payments of value P_1 until age 65. However, in that case, the benefit is reduced from the unit to some lower amount B that is paid for death after the age 35. Calculate B.
4. If the option in 3. is chosen, calculate the twentieth-year reserve.

Present your derivations in your written solution as well.

You do not have to create Word/LaTex/... form of your solution. A handwritten (but legible) solution with added graphs is completely fine.

Send your solution as a PDF file *surname_name_HW2.pdf* to vejmelp@karlin.mff.cuni.cz until January 7, 2024.