Mathematics of Life Insurance 1 - HW2

Part 1 (4 points)

Consider the data published by the Czech Statistical Office for the whole Czech Republic for year 2023: link to data. Create your own life tables in Excel for **men / women / unisex**, where you begin only with the knowledge of l_x (table number of male survivors) and l_y (table number of female survivors). Compute (d_x, p_x, q_x) and commutation functions $(C_x, D_x, M_x, N_x, R_x, S_x)$ for TIR $= i = 1\%$. For preparing unisex tables use $l_{uni} = 0.5 \cdot l_x + 0.5 \cdot l_y$.

When the commutation functions are calculated, plot the net single premiums for the following capital life insurances for ages (x from 20 to 60 per 1 year) and all three life tables (men, women, unisex).

1. Pure endowment for $n = 65 - x$ years,
2. Standard increasing term insurance until age 65, i.e., for $n = 65 - x$ years.

In the pdf, tables with the calculated commutation functions for all three cases should be included. Then for each insurance give one graph with three lines with respect to used life tables and add a short comment explaining the behaviour of the net single premiums.

Part 2 (3 points)

Consider a life aged 50 and a special annually decreasing 5-year term insurance paying 5,000 at the moment of death in the first year, 4,000 in the second year, and so on. Use **unisex** life tables constructed in Part 1 together with i considered there and assume a uniform distribution of deaths over each year.
Calculate the net present value of this insurance.

Part 3 (4 points)

Derive an expression for $(\overline{IA})_x = \mathrm{E}(T \cdot v^T)$, when given $\mu_{x+t} = \mu$ and $\delta_t = \delta$ for all $t > 0$.

Part 4 (4 points)

Suppose that a life aged 30 arranged an insurance with the following parameters.
If their death occurs during the first 20 years, 10,000 is paid; otherwise, 20,000 is paid (at time 20). Moreover, it was arranged with the insurance company that the SI will be increased by 25% of the **net single premium** in the case of death during the first 5 years.
Assume that it holds: $l_x = 100 - x, \quad 0 \leq x \leq 100$ and $i = 0.05$.
Calculate the net single premium of this insurance.

You do not have to create a Word/LaTex/... form of your solution. Sending a handwritten (but legible) text is fine. Then just attach graphs or tables to this solution.

Send your solution as a PDF file *surname_name_HW2.pdf* to vejmelp@karlin.mff.cuni.cz until December 1, 2024.