Mathematics of Life Insurance 1 - Sample test

Part 1 (3 points)

$$1000 = \frac{x}{1.01} + \frac{x}{1.01^2} + \frac{x}{1.01^3} + \frac{3x}{1.01^4} + \frac{3x}{1.01^5} + \frac{3x}{1.01^6}$$
$$1000 = \left[\frac{1}{1.01} + \frac{1}{1.01^2} + \frac{1}{1.01^3} + \frac{3}{1.01^4} + \frac{3}{1.01^5} + \frac{3}{1.01^6}\right] \cdot x$$
$$1000 = 11.5045 \cdot x$$
$$x = 86.9228$$

Part 2 (3 points)

$$\mu_{x+t} = \frac{1}{85-t} + \frac{3}{105-t}, \qquad 0 \le t < 85$$
$${}_tp_x = \exp\left(-\int_0^t \mu_{x+s}\, ds\right)$$
$${}_{20}p_x = \exp\left(-\int_0^{20} \frac{1}{85-s}\, ds - \int_0^{20} \frac{3}{105-s}\, ds\right)$$
Substitution: $85 - s = y \Rightarrow -ds = dy$ and $105 - s = z \Rightarrow -ds = dz$
$${}_{20}p_x = \exp\left(\int_{85}^{65} \frac{1}{y}\, dy + 3 \cdot \int_{105}^{85} \frac{1}{z}\, dz\right)$$
$${}_{20}p_x = \exp\left[\ln(65) - \ln(85) + 3 \cdot \ln(85) - 3 \cdot \ln(105)\right] = 0.4057$$

Part 3 (3 points)

$$\boxed{Z = \begin{cases} 0, & K = 0, 1, \ldots, m-1 \\ v^{K+1}, & K = m, m+1, \ldots, m+n-1 \\ v^{m+n}, & K = m+n, m+n+1, \ldots \end{cases}}$$

$${}_{m|}A_{x:\overline{n}|} = \sum_{k=m}^{m+n-1} \left[v^{k+1} \cdot {}_kp_x \cdot q_{x+k}\right] + v^{m+n} \cdot {}_{m+n}p_x$$

Part 4 (3 points)

$$\ddot{a}_{\overline{K+1}|} = \frac{1 - v^{K+1}}{1 - v} = \frac{1 - v^{K+1}}{d}$$
$$\mathrm{E}(\ddot{a}_{\overline{K+1}|}) = \frac{1 - \mathrm{E}(v^{K+1})}{d}$$
$$\ddot{a}_x = \frac{1 - A_x}{d}$$
$$A_x = 1 - d \cdot \ddot{a}_x$$

Part 5 (3 points)

$$\boxed{Y = \begin{cases} 1 + 2 \cdot v + \cdots + (K+1) \cdot v^K, & K = 0, 1, \ldots, n-1 \\ 1 + 2 \cdot v + \cdots + n \cdot v^{n-1}, & K = n, n+1, \ldots \end{cases}}$$

$$(I\ddot{a})_{x:\overline{n}|} = \sum_{k=0}^{n-1} \left[(k+1) \cdot v^k \cdot {}_kp_x \right]$$

Part 6 (5 points)

$$\boxed{L = \begin{cases} 0 + 0.8 \cdot P \cdot (K+1) \cdot v^{K+1} - P \cdot \sum_{k=0}^{K} v^k, & K = 0, 1, \ldots, m^{'} - 1 \\ 0 - P \cdot \sum_{k=0}^{K} v^k, & K = m^{'}, m^{'} + 1, \ldots, m-1 \\ v^{K+1} - P \cdot \sum_{k=0}^{m-1} v^k, & K = m, \ldots, m+n-1 \\ 0 - P \cdot \sum_{k=0}^{m-1} v^k, & K = m+n, m+n+1 \ldots \end{cases}}$$

$$EL = 0 = {}_{m|}A^1_{x:\overline{n}|} + 0.8 \cdot P \cdot (IA)_{\overline{1}\ \overline{x:m^{'}}|} - P \cdot \ddot{a}_{x:\overline{m}|}$$

$$\Downarrow$$

$$P = \frac{{}_{m|}A^1_{x:\overline{n}|}}{\ddot{a}_{x:\overline{m}|} - 0.8 \cdot (IA)_{\overline{1}\ \overline{x:m^{'}}|}}$$

$$\boxed{{}_kV_x = \begin{cases} {}_{m-k|}A^{\overline{1}}_{x+k:\overline{n}|} + 0.8 \cdot P \cdot (IA)_{\overline{1}\ \overline{x+k:m^{'}-k}|} + 0.8 \cdot P \cdot k \cdot A_{\overline{1}\ \overline{x+k:m^{'}-k}|} & \\ -P \cdot \ddot{a}_{x+k:\overline{m-k}|}, & k = 0, 1, \ldots, m^{'} - 1 \\ {}_{m-k|}A^{\overline{1}}_{x+k:\overline{n}|} - P \cdot \ddot{a}_{x+k:\overline{m-k}|}, & k = m^{'}, \ldots, m-1 \\ A^{\overline{1}}_{x+k:\overline{n+m-k}|}, & k = m, \ldots, m+n-1 \end{cases}}$$