Mathematics of Life Insurance 1

Final test - Solution

Part 1 (2 points)

$$PV = \sum_{t=0}^{14} 200 \cdot \left(\frac{1}{1+0.135}\right)^t = 200 \cdot \frac{1-v^{15}}{1-v} = 1,429.9\$.$$

Part 2 (3 points)

$$q_x = 1 - p_x; \qquad q_x^* = \frac{q_x}{2} = \frac{1-p_x}{2}$$

$$p_x = \exp\left(-\int_0^1 \mu_{x+s}\,ds\right); \qquad p_x^* = \exp\left(-\int_0^1 \mu_{x+s} - c\,ds\right) = p_x \cdot e^c$$

$$1 - \frac{1-p_x}{2} = p_x \cdot e^c \qquad \to \qquad \frac{1+p_x}{2} = p_x \cdot e^c$$

$$e^c = \frac{1+p_x}{2 \cdot p_x} = \frac{2-q_x}{2 \cdot (1-q_x)}$$

$$c = \ln\left(\frac{2-q_x}{2 \cdot (1-q_x)}\right)$$

Part 3 (3 points)

$${}_kp_x = \frac{l_{x+k}}{l_x}; \qquad {}_kq_x = \frac{l_x - l_{x+k}}{l_x}; \qquad C_x = (l_x - l_{x+1}) \cdot v^{x+1}; \qquad D_x = l_x \cdot v^x$$

$$\boxed{Z = \begin{cases} 100,000 \cdot v^{K+1}, & K = 0, \ldots, 9 \\ (100,000 - 10,000 \cdot (K-9))v^{K+1}, & K = 10, \ldots, 18 \\ 0, & K = 19, 20, \ldots \end{cases}}$$

$$NSP = \sum_{k=0}^{9} \left(100,000 \cdot v^{k+1} \cdot {}_kp_x \cdot q_{x+k}\right) + \sum_{k=10}^{18} \left((100,000 - 10,000 \cdot (k-9)) \cdot v^{k+1} \cdot {}_kp_x \cdot q_{x+k}\right)$$

Part 4 (4 points)

Assumption of linearity: $q_{x+u} = u \cdot q_x$ for $u \in (0,1)$

$$\overline{A}_x = \frac{i}{\delta} \cdot A_x$$

$$\overline{A}_x = E[v^T] = E[v^{K+S}] = E[v^{(K+1)+(S-1)}] = E[v^{K+1}] \cdot E[v^{S-1}] = E[v^{S-1}] \cdot A_x$$

$$E[v^{S-1}] = \int_0^1 v^{s-1}\,dt = \left[\frac{v^{s-1}}{\ln(v)}\right]_0^1 = \frac{1}{\ln(v)} - \frac{1}{v \cdot \ln(v)} = \frac{v-1}{v \cdot \ln(v)} = \frac{\frac{-i}{1+i}}{\frac{-\delta}{1+i}} = \frac{i}{\delta}$$

Part 5 (4 points)

$$\boxed{L = \begin{cases} P \cdot (K+1) \cdot v^{K+1} - P \cdot \sum_{k=0}^{K} v^k, & K = 0, \ldots, 9 \\ 0 - P \cdot \sum_{k=0}^{K} v^k, & K = 10, \ldots, 19 \\ \sum_{k=20}^{K} v^k - P \cdot \sum_{k=0}^{19} v^k, & K = 20, \ldots, 49 \\ \sum_{k=20}^{49} v^k - P \cdot \sum_{k=0}^{19} v^k, & K = 50, 51, \ldots \end{cases}}$$

$$\mathbb{E}L = 0 = {}_{20|}\ddot{a}_{x:\overline{30|}} + P \cdot (IA)^{1}_{x:\overline{10|}} - P \cdot \ddot{a}_{x:\overline{20|}}$$

$$\Downarrow$$

$$P = \frac{{}_{20|}\ddot{a}_{x:\overline{30|}}}{\ddot{a}_{x:\overline{20|}} - (IA)^{1}_{x:\overline{10|}}}$$

Part 6 (4 points)

$$\mathbb{E}L = 0 = {}_{m|}A_{x:\overline{n|}} - P \cdot \ddot{a}_{x:\overline{m|}}$$

$$\Downarrow$$

$$P = \frac{{}_{m|}A_{x:\overline{n|}}}{\ddot{a}_{x:\overline{m|}}}$$

$$\boxed{{}_kV_x = \begin{cases} {}_{m-k|}A_{x+k:\overline{n|}} - P \cdot \ddot{a}_{x+k:\overline{m-k|}}, & k = 0, \ldots, m-1 \\ A_{x+k:\overline{n+m-k|}}, & k = m, \ldots, m+n-1 \end{cases}}$$