Mathematics of Life Insurance 2 - HW1

Part 1 (5 points)

Consider some kind of insurance with a total loss defined as $L = T \cdot v^T - \Pi \cdot \overline{a}_{\overline{T}|}$ with constant forces of mortality and interest. Express Π and ${}_t\overline{V}$ in terms of μ and δ.

Part 2 (4 points)

Consider the continuous model and assume that $c_t = {}_tV_x$, ${}_0V_x = 0$ and $\Pi_t = \Pi$ for $t \geq 0$. Show that ${}_tV_x = \Pi \cdot \overline{s}_{\overline{t}|}$, where $\overline{s}_{\overline{t}|} = \frac{e^{\delta \cdot t} - 1}{\delta}$.

Part 3 (6 points)

A 10,000 whole life policy is issued to a life aged 30 based on the unisex life tables. The actual interest earned in policy years 1-5 is $i' = 2.7\%$. Assume the policyholder is alive at age 35 and the policy is in force.

1. Calculate the technical gain realised in each year using method (2) presented during lectures (with the use of gain from savings and gain from insurance).
2. Calculate the accumulated value of the gains (using $i' = 2.7\%$) at age 35.
3. Determine the value of i' (level over five years) for which the accumulated gains are equal to 200.

Part 4 (5 points)

Let $J = 1$ represent death by accidental means and $J = 2$ death by other means. Assume that $\delta = 0.05$, $\mu_{1,x+t} = 0.005$ for $t \geq 0$, where $\mu_{1,x+t}$ is the force of decrement for death by accidental means and $\mu_{2,x+t} = 0.02$ for $t \geq 0$.
A 20-year term insurance policy, payable at the moment of death, is issued to a life age x providing a benefit of 2 if death is by accidental means and providing a benefit of 1 for other deaths.
Find the expectation and variance of the present value of benefits random variable.

You do not have to create Word/LaTex/... form of your solution. A handwritten (but legible) solution with added graphs is completely fine.

Send your solution as a PDF file *surname_name_HW1.pdf* together with an Excel file for Part 3 to vejmelp@karlin.mff.cuni.cz until March 28, 2024.