

Mathematics of Life Insurance 2 - HW2

Part 1 (5 points)

Assume the following compound model with one decrement:

x	$q'_{1,x}$	$q'_{2,x}$	$q'_{3,x}$
65	0.020	0.02	0.04
66	0.025	0.02	0.06
67	0.030	0.02	0.08
68	0.035	0.02	0.10
69	0.040	0.02	0.12

1. Under the assumption of linearity for each cause of decrement compute $q_{1,x}$, $q_{2,x}$ and $q_{3,x}$.
2. Now, assume that columns $q'_{1,x}$, $q'_{2,x}$ and $q_{3,x}$ are known. Derive $q'_{3,x}$.

Hint for task 2.: Realize that $q_{j,x} = \int_0^1 {}_t p_x \cdot \mu_{j,x+t} dt$. Use the ass. of linearity to get rid of $\mu_{j,x+t}$.

Part 2 (5 points)

Consider random variables T_x and T_y that have the joint probability density function in the following form

$$f_{T_x T_y}(s, t) = \frac{(n-1)(n-2)}{(1+s+t)^n}, \quad s > 0, t > 0, n > 2.$$

Derive the prob. density function and distribution function of $T_{x:y}$, $E(T_{x:y})$ for $n > 3$ and $\mu_{x+t:y+t}$.

Part 3 (4 points)

Use the life tables that you created in the winter semester and the assumed $i = 2\%$ to calculate the joint life annuity $a_{x:y}$ and the last survivor annuity $a_{\overline{x:y}}$ for independent lives age $x = 65$ (male) and $y = 60$ (female).

Part 4 (6 points)

Compute and plot the gross premium reserves for

1. the endowment insurance until 70 years with the net annual premium collected during the whole period with SI=1,000,000,
2. the life annuity in advance deferred until the age 70 years with the net annual premium collected during the deferment period with SI=120,000.

Consider the input ages $x = 25, 30$ and 35 years. Use the unisex life tables (TIR=2%) and the corresponding commutation functions. The expenses values are

- $\alpha = 5\%$
- $\beta = 0.8\%$ (collection expenses)
- $\gamma = 2\%$ (collected over the entire contract period, even when annuity payments are made)
- $\delta = 0.5\%$

Add a few words about the obtained results and include also the general formulas for the premium and reserves in your solution.

Send a PDF file *surname_name_HW2.pdf* to vejmel@karlin.mff.cuni.cz until **April 30, 2024**.