Mathematics of Life Insurance 2

Sample Test - Solution

Part 1 (3 points)

$$\begin{aligned} EZ &= \frac{2}{3}\bar{a}_{\overline{x:y}} + \frac{1}{3}\bar{a}_{x:y} = \frac{2}{3}\int_0^\infty v^t \cdot {}_tp_{\overline{x:y}}\,dt + \frac{1}{3}\int_0^\infty v^t \cdot {}_tp_{x:y}\,dt \\ &= \frac{2}{3}\int_0^\infty v^t \cdot {}_tp_x\,dt + \frac{2}{3}\int_0^\infty v^t \cdot {}_tp_y\,dt - \frac{1}{3}\int_0^\infty v^t \cdot {}_tp_{x:y}\,dt \\ &= \frac{2}{3}\int_0^\infty v^t \cdot {}_tp_x\,dt + \frac{2}{3}\int_0^\infty v^t \cdot {}_tp_y\,dt - \frac{1}{3}\int_0^\infty v^t \cdot {}_tp_x \cdot {}_tp_y\,dt \end{aligned}$$

Part 2 (3 points)

The (generalized) equivalence principle is

$$P^B \cdot \ddot{a}_{x:\overline{m}|} = (IA)^1_{x:\overline{n}|} + \alpha + \beta \cdot P^B \cdot \ddot{a}_{x:\overline{m}|} + \gamma \cdot \ddot{a}_{x:\overline{n}|},$$

and therefore,

$$P^B = \frac{(IA)^1_{x:\overline{n}|} + \alpha + \gamma \cdot \ddot{a}_{x:\overline{n}|}}{(1-\beta)\cdot \ddot{a}_{x:\overline{m}|}}.$$

The components of the expense-loaded premium reserve are:
the net premium reserve

$$\boxed{{}_kV_x = \begin{cases} (IA)^1_{x+k:\overline{n-k}|} + k \cdot A^1_{x+k:\overline{n-k}|} - P \cdot \ddot{a}_{x+k:\overline{m-k}|}, & k = 0,\dots,m-1, \\ (IA)^1_{x+k:\overline{n-k}|} + k \cdot A^1_{x+k:\overline{n-k}|}, & k = m,\dots,n-1 \end{cases}}$$

the reserve for the acquisition expenses

$$\boxed{{}_kV_x^\alpha = \begin{cases} I(k=0)\cdot\alpha - P^\alpha \cdot \ddot{a}_{x+k:\overline{m-k}|}, & k = 0,\dots,m-1, \\ 0, & k = m,\dots,n-1 \end{cases}}$$

the reserve for the collection expenses

$$\boxed{{}_kV_x^\beta = 0}$$

the reserve for the administration expenses

$$\boxed{{}_kV_x^\gamma = \begin{cases} \gamma \cdot \ddot{a}_{x+k:\overline{n-k}|} - P^\gamma \cdot \ddot{a}_{x+k:\overline{m-k}|}, & k = 0,\dots,m-1, \\ \gamma \cdot \ddot{a}_{x+k:\overline{n-k}|}, & k = m,\dots,n-1 \end{cases}}$$

Then we have

$$\boxed{{}_kV_x^B = {}_kV_x + {}_kV_x^\alpha + {}_kV_x^\beta + {}_kV_x^\gamma, \qquad k = 0,\dots,n-1}$$

Part 3 (3 points)

$$\mu_{j,x+t} = \frac{j}{150}, \qquad j = 1,2,3$$

$$E[T|J=3] = \int_0^\infty t \cdot \frac{g_3(t)}{P(J=3)}\,dt = \frac{\int_0^\infty t \cdot g_3(t)\,dt}{P(J=3)}$$

$$\int_0^\infty t \cdot g_3(t)\,dt = \int_0^\infty t \cdot {}_tp_x \cdot \mu_{3,x+t}\,dt$$

$$ {}_tp_x = exp\{-\int_0^t \sum_{j=1}^{3} \mu_{j,x+s}\, dt\} = exp\{-\int_0^t \frac{6}{150}\, dt\} = exp\{-\frac{t}{25}\} $$
$$ \int_0^\infty t \cdot {}_tp_x \cdot \mu_{3,x+t}\, dt = \int_0^\infty t \cdot \frac{3}{150} \cdot exp\{-\frac{t}{25}\}\, dt = \frac{1}{2} \cdot \int_0^\infty t \cdot \frac{1}{25} \cdot exp\{-\frac{t}{25}\}\, dt = \frac{25}{2} $$
$$ P(J=3) = \int_0^\infty g_3(t)\, dt = \int_0^\infty \frac{3}{150} \cdot exp\{-\frac{t}{25}\} = \frac{1}{2} \cdot \int_0^\infty \frac{1}{25} \cdot exp\{-\frac{t}{25}\}\, dt = \frac{1}{2} $$
$$ E[T|J=3] = 25 $$

Part 4 (2 points)

$$ {}_kV_x = \sum_{j=0}^{\infty} c_{k+j+1} \cdot v^{j+1} \cdot {}_jp_{x+k} \cdot q_{x+k+j} - \sum_{j=0}^{\infty} \Pi_{k+j} \cdot v^j \cdot {}_jp_{x+k} $$
$$ c_1 = c_2 = \ldots = 0 $$
$$ \Pi_0 = P = {}_{m|}a_x, \qquad \Pi_1 = \ldots = \Pi_m = 0, \qquad \Pi_{m+1} = \Pi_{m+2} = \ldots = -1 $$

Part 5 (2 points)

Decrements: $J = 1 \ldots$death; $J = 2 \ldots$disablement; $m = 2$

$$ \text{NSP} = \sum_{j=1}^{m} \sum_{k=0}^{\infty} c_{j,k+1} \cdot v^{k+1} \cdot {}_kp_x \cdot q_{j,x+k} - \sum_{k=0}^{\infty} \pi_k \cdot v^k \cdot {}_kp_x. $$

In our case, we set

- $c_{1,k+1} = \begin{cases} 5 \cdot 10^6, & k = 0, \ldots, 39, \\ 0, & k \geq 40, \end{cases}$
- $c_{2,k+1} = \begin{cases} 350,000 \cdot \ddot{a}_{x+k+1}, & k = 0, \ldots, 44, \\ 0, & k \geq 45, \end{cases}$
- $\pi_k = \begin{cases} -240,000 \cdot \ddot{a}_{x+45} - 10^4 \cdot (I\ddot{a})_{x+45}, & k = 45, \\ 0, & \text{otherwise.} \end{cases}$

Part 6 (2 point)

$$ \Pi(t) = \Pi^r(t) + \Pi^s(t), $$

where

$$ \Pi^r(t) = (c(t) - V(t)) \cdot \mu_{x+t}. $$

Therefore, $\Pi^r(t) = (1 - 0.5) \cdot 0.1 = 0.05$ and $\Pi(t) = 0.05 + 0.03 = 0.08$.