Mathematics of Life Insurance 2 - Sample test - Solution

Part 1 (1 point)

$$
\begin{aligned}
\mathrm{P}(5 < T \le 10) &= \mathrm{P}(T > 5) - \mathrm{P}(T > 10) = {}_5p_{\overline{80:85}} - {}_{10}p_{\overline{80:85}} \\
&= {}_5p_{80} + {}_5p_{85} - {}_5p_{80:85} - {}_{10}p_{80} - {}_{10}p_{85} + {}_{10}p_{80:85} \\
&= {}_5p_{80} + {}_5p_{85} - {}_5p_{80} \cdot {}_5p_{85} - {}_{10}p_{80} - {}_{10}p_{85} + {}_{10}p_{80} \cdot {}_{10}p_{85}
\end{aligned}
$$

Part 2 (3 points)

The (generalized) equivalence principle is

$$
P^B \cdot \ddot{a}_{x:\overline{m}|} = {}_{m|}(I\ddot{a})_{x:\overline{n}|} + \alpha + \beta \cdot P^B \cdot \ddot{a}_{x:\overline{m}|} + \gamma \cdot \ddot{a}_{x:\overline{m}|} + \delta \cdot {}_{m|}(I\ddot{a})_{x:\overline{n}|},
$$

and therefore,

$$
P^B = \frac{(1+\delta) \cdot {}_{m|}(I\ddot{a})_{x:\overline{n}|} + \alpha + \gamma \cdot \ddot{a}_{x:\overline{m}|}}{(1-\beta) \cdot \ddot{a}_{x:\overline{m}|}}.
$$

The components of the expense-loaded premium reserve are:
the net premium reserve

$$
\boxed{{}_kV_x = \begin{cases} {}_{m-k|}(I\ddot{a})_{x+k:\overline{n}|} - P \cdot \ddot{a}_{x+k:\overline{m-k}|}, & k = 0, \ldots, m-1, \\ (I\ddot{a})_{x+k:\overline{n+m-k}|} + (k-m) \cdot \ddot{a}_{x+k:\overline{n+m-k}|}, & k = m, \ldots, m+n-1 \end{cases}}
$$

the reserve for the acquisition expenses

$$
\boxed{{}_kV_x^{\alpha} = \begin{cases} I(k=0) \cdot \alpha - P^{\alpha} \cdot \ddot{a}_{x+k:\overline{m-k}|}, & k = 0, \ldots, m-1, \\ 0, & k = m, \ldots, m+n-1 \end{cases}}
$$

the reserve for the collection expenses

$$
\boxed{{}_kV_x^{\beta} = 0}
$$

the reserve for the administration expenses

$$
\boxed{{}_kV_x^{\gamma} = 0}
$$

and the reserve for the annuity expenses

$$
\boxed{{}_kV_x^{\delta} = \begin{cases} \delta \cdot {}_{m-k|}(I\ddot{a})_{x+k:\overline{n}|} - P^{\delta} \cdot \ddot{a}_{x+k:\overline{m-k}|}, & k = 0, \ldots, m-1, \\ \delta \cdot (I\ddot{a})_{x+k:\overline{n+m-k}|} + \delta \cdot (k-m) \cdot \ddot{a}_{x+k:\overline{n+m-k}|}, & k = m, \ldots, m+n-1 \end{cases}}
$$

Then we have

$$
\boxed{{}_kV_x^B = {}_kV_x + {}_kV_x^{\alpha} + {}_kV_x^{\beta} + {}_kV_x^{\gamma} + {}_kV_x^{\delta}, \qquad k = 0, \ldots, m+n-1}
$$

Part 3 (1 point)

$$
\Pi(t) = \Pi^r(t) + \Pi^s(t),
$$

where

$$
\Pi^r(t) = (c(t) - V(t)) \cdot \mu_{x+t}.
$$

Therefore, $\Pi^r(t) = (1-0.5) \cdot 0.1 = 0.05$ and $\Pi(t) = 0.05 + 0.03 = 0.08$.

Part 4 (2 points)

$$L = \begin{cases} v^{K+1} + 0.7 \cdot P \cdot (K+1) \cdot v^{K+1} - P \cdot \sum_{k=0}^{K} v^k, & K = 0, 1, \ldots, n-1 \\ v^n - P \cdot \sum_{k=0}^{n-1} v^k, & K = n, n+1, \ldots \end{cases}$$

$$\mathbb{E}L = 0 = A_{x:n} + 0.7 \cdot P \cdot (IA)^1_{x:\overline{n}|} - P \cdot \ddot{a}_{x:\overline{n}|}$$

$$\Downarrow$$

$$P = \frac{A_{x:n}}{\ddot{a}_{x:\overline{n}|} - 0.7 \cdot (IA)^1_{x:\overline{n}|}}$$

Part 5 (2 points)

Decrements: $J = 1 \ldots$death; $J = 2 \ldots$disablement; $m = 2$

$$\text{NSP} = \sum_{j=1}^{m} \sum_{k=0}^{\infty} c_{j,k+1} \cdot v^{k+1} \cdot {}_kp_x \cdot q_{j,x+k} - \sum_{k=0}^{\infty} \pi_k \cdot v^k \cdot {}_kp_x.$$

In our case, we set

- $c_{1,k+1} = \begin{cases} 5 \cdot 10^6, & k = 0, \ldots, 39, \\ 0, & k \geq 40, \end{cases}$
- $c_{2,k+1} = \begin{cases} 350,000 \cdot \ddot{a}_{x+k+1}, & k = 0, \ldots, 44, \\ 0, & k \geq 45, \end{cases}$
- $\pi_k = \begin{cases} -240,000 \cdot \ddot{a}_{x+45} - 10^4 \cdot (I\ddot{a})_{x+45}, & k = 45, \\ 0, & \text{otherwise.} \end{cases}$

Part 6 (1 point)

1. Compute $p'_{j,x} = 1 - q'_{j,x}$ for all j.
2. Derive
$$p_x = \prod_{j=1}^{m} p'_{j,x} \text{ and } q_x = 1 - p_x.$$
3. Apply the formula
$$q_{j,x} = q_x \cdot \frac{\ln p'_{j,x}}{\ln p_x}.$$

This is valid under the assumption of linearity or under the assumption of constant force of mortality. e