How to choose? Preferences, Utility and Stochastic Dominance

Sebastiano Vitali

Charles University, MFF
Praha

Does it ever happen that we need to make a choice?

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Every day

Do you run an optimization model to make your choice?

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Typically not...

But still, is your decision somehow rational?

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Hopefully...

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We try to tackle these issues during this presentation

- **Risk**: situation where both possible outcomes and their probabilities are known (models)

Risk vs Uncertainty

- **Risk**: situation where both possible outcomes and their probabilities are known (models)

- **Uncertainty**: situation either where possible outcomes or their probabilities are **not** known (life)

Economic agents are assumes to behave according to their preferences.

Two main relationships are used to describe preferences:

- "to be preferred to" ($\succeq$): when the payoffs represented by vector x are preferred to payoffs in vector y

$$x \succeq y$$

- "to be indifferent to" ($\sim$): when the payoffs represented by vector x are indifferent to payoffs in vector y

$$x \sim y$$

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Actually

$$x \succeq y \wedge y \succeq x \Rightarrow x \sim y \tag{1}$$

So we can just study the $\succeq$ relation and the other is only a sub-case.

An economic agent is said to be **rationale** if her preferences are

- **complete**: an agent is always able to define her preferences when facing a choice (strong condition). This means that

$$\forall x, y \quad \text{either } x \succeq y \vee y \succeq x \vee x \sim y \text{ is true} \tag{2}$$

- **transitive**: given three vectors x, y, z

$$x \succeq y \wedge y \succeq z \Rightarrow x \succeq z \tag{3}$$

- **continue**: given three vectors x, y, z

$$\forall x \succeq y \succeq z \Rightarrow \exists \lambda \in [0, 1] : \lambda x + (1 - \lambda) z \sim y \tag{4}$$

Preferences (no Risk)

Theorem

If preferences are rational (complete and transitive), and continuous, then there exist a (continuous) function $U(\cdot)$ (so-called "utility function") such that

$$x \succeq y \Leftrightarrow U(x) \geq U(y)$$

Of course, the utility function is not unique. Any increasing function does not alter the inequality. For any $V(\cdot)$ increasing:

$$U(x) \geq U(y) \Leftrightarrow V(U(x)) \geq V(U(y))$$

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Utility functions are good for ordering,
not for measuring!

Further properties for a reasonable utility function $U(\cdot)$. Utility is

- **increasing**: an agent always prefers more to less, i.e.

$$\frac{\partial U(x)}{\partial x} > 0 \tag{5}$$

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- **concave**: the marginal utility is decreasing

$$\frac{\partial^2 U(x)}{\partial x^2} < 0 \tag{6}$$

Preferences (with Risk)

Previous theorem is useful to study situation without risk. In presence of risk we need another assumption:

- **independence**: if an agent prefers x to y and must chose between two bundles (x, z) and (y, z) containing z in the same proportion, then she will chose (x, z).

$$x \succeq y \Leftrightarrow \forall z, \lambda \in [0, 1] : \lambda x + (1 - \lambda)z \succeq \lambda y + (1 - \lambda)z \quad (7)$$

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Expected utility

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x	$=$	$\{$5mil.	1
y	$=$	$\{$ 0	0.01
		5mil.	0.89
		25mil.	0.1

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Expected utility can fail!!

But still, in most cases, we can say something useful with utilities. For instance we can interpret the **risk attitude** of an investor.

Assume there are two possible outcomes W_1 and W_2 having respectively utility $U(W_1)$ and $U(W_2)$. The expected utility lines on the straight line between the two points (see figure!), what about the utility of the combination between W_1 and W_2?

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The aversion can change with the wealth level, then we introduce the Arrow-Pratt (**R**elative) **R**isk **A**version:

$$RRA = -\frac{U''(W)W}{U'(W)}$$

Then, we can distinguish:

CARA	Constant	DARA	Decreasing
IARA	Increasing	HARA	Hyperbolic

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Then, fixing values for α, β and γ, we obtain all the classes.

CARA $\alpha = 1$, $\gamma = 0$

IARA $\gamma = -\beta$

DARA $\alpha = 1$ $\gamma = 1$

HARA any

Another experiments

Assume you invest 100 and you can get the following equiprobable realizations.

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$$\begin{array}{ccc} 98 & \geq & 98 \\ 105 & \geq & 105 \\ 110 & \geq & 107 \end{array} \qquad \text{cum}(x) = \begin{bmatrix} 90 \\ 195 \\ 305 \end{bmatrix}, \quad \text{cum}(y) = \begin{bmatrix} 98 \\ 203 \\ 310 \end{bmatrix}$$

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Second order dominance!

First order Stochastic Dominance

Given two distributions X and Y we define that X **first-order stochastically dominates** Y *if*

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If the random variables are discrete and taking n values each with probability $1/n$: $\mathbf{X} \geq \mathbf{P} \cdot \mathbf{Y}$, where $\mathbf{P}$ is a permutation matrix.

Defining the twice cumulative distribution function as

$$F_X^{(2)}(\eta) = \int_{-\infty}^{\eta} F_X(\alpha) d\alpha \quad (14)$$

Given two distributions X and Y we define that X **second-order stochastically dominates** Y *if*

$$F_X^{(2)}(\eta) \leq F_Y^{(2)}(\eta), \forall \eta \in \Re \quad (15)$$

Second order Stochastic Dominance

Defining the twice cumulative distribution function as

$$F_X^{(2)}(\eta) = \int_{-\infty}^{\eta} F_X(\alpha) d\alpha \tag{14}$$

Given two distributions X and Y we define that X **second-order stochastically dominates** Y *if*

$$F_X^{(2)}(\eta) \leq F_Y^{(2)}(\eta), \forall \eta \in \Re \tag{15}$$

Moreover, X **second-order stochastically dominates** Y *if and only if* every expected utility maximizer with an **nondecreasing and concave utility function** prefers X over Y: $\mathbb{E}[U(X)] \geq \mathbb{E}[U(Y)], \forall U \in \mathcal{U}_2$, where $\mathcal{U}_2$ is the set of all nondecreasing and concave utility functions.

Defining the twice cumulative distribution function as

$$F_X^{(2)}(\eta) = \int_{-\infty}^{\eta} F_X(\alpha) d\alpha \quad (14)$$

Given two distributions X and Y we define that X **second-order stochastically dominates** Y *if*

$$F_X^{(2)}(\eta) \leq F_Y^{(2)}(\eta), \forall \eta \in \Re \quad (15)$$

Moreover, X **second-order stochastically dominates** Y *if and only if* every expected utility maximizer with an **nondecreasing and concave utility function** prefers X over Y: $\mathbb{E}[U(X)] \geq \mathbb{E}[U(Y)], \forall U \in \mathcal{U}_2$, where $\mathcal{U}_2$ is the set of all nondecreasing and concave utility functions.
If the random variables are discrete and taking n values each with probability $1/n$: $\mathbf{X} \geq \mathbf{W} \cdot \mathbf{Y}$, where $\mathbf{W}$ is a double stochastic matrix.

Conclusions

Observation

Conclusions

Observation

Reason

Observation

Reason

Consequence

Conclusions

Observation For daily life decisions you face uncertainty, not risk

Reason

Consequence

Conclusions

Observation For daily life decisions you face uncertainty, not risk

Reason you **do not** know the probability

Consequence

Conclusions

Observation For daily life decisions you face uncertainty, not risk

Reason you **do not** know the probability

Consequence use utility for preferences!!

Conclusions

Observation For model decisions you face risk

Reason

Consequence

Conclusions

Observation For model decisions you face risk

Reason you **do** know the probability

Consequence

Conclusions

Observation For model decisions you face risk

Reason you **do** know the probability

Consequence use **expected** utility for preferences!!

Observation The agent has some attitude

Reason

Consequence

Conclusions

Observation The agent has some attitude

Reason risk-averse, risk-neutral, risk-lover, ...

Consequence

Conclusions

Observation The agent has some attitude

Reason risk-averse, risk-neutral, risk-lover, ...

Consequence use **appropriate** utility function!!

Conclusions

Observation To choose the utility function could be challenging

Reason

Consequence

Observation To choose the utility function could be challenging

Reason each risk-attitude may have multiple utility functions

Consequence

Conclusions

Observation To choose the utility function could be challenging

Reason each risk-attitude may have multiple utility functions

Consequence use **stochastic dominance**!!

Bibliography

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