1 Consumer theory and its applications

1.1 Preferences and utility

1.2 Utility maximization and uncompensated demand

1.3 Expenditure minimization and compensated demand

1.4 Price changes and welfare

1.5 Labour supply, taxes and benefits

1.6 Saving and borrowing
2 Firms, costs and profit maximization

2.1 Firms and costs

2.2 Profit maximization and costs for a price taking firm

3. Industrial organization

3.1 Perfect competition and monopoly

3.2 Oligopoly and games
1.4 Price changes and welfare
1.4 Price changes and welfare

1. Price indices
2. Substitution bias
3. Compensating variation (CV)
4. Equivalent variation (EV)
5. Compensating variation vs Equivalent variation
6. Compensating variation, equivalent variation and change in consumer surplus
7. Income effects, CV, EV and consumer surplus

8. Using EV to assess the effect of a tax

9. Using EV to assess the effect of a subsidy

9. Does compensating a consumer for a price increase imply that the price increase has no effect on demand?

10. Benefits in kind
Base weighted price indices
1. Price indices measure inflation

- Public sector use
  - Inflation targeting
  - Adjusting levels of taxes, benefits, public pensions
  - Indexed government bonds
  - Measurement of real wages

- Private sector use
  - Pensions
  - Measurement of real wages
  - Price & wage setting
The design of price indices matters and is controversial

- Prices of different things change at different rates.
- Price indices are weighted averages
  - What should be included in the index?
  - Weights
  - Formula
1. Price Indices

Base weighted price index

Also called **Laspeyres** Price Index

CPI index is usually a base weighted index

\[ x_{1A}, x_{2A}, \ldots, x_{nA} \] consumption at date A, index is

\[
\frac{p_{1B}x_{1A} + p_{2B}x_{2A} + \ldots + p_{nB}x_{nA}}{p_{1A}x_{1A} + p_{2A}x_{2A} + \ldots + p_{nA}x_{nA}}
\]
Base weighted price index

\[ p_{1B}x_{1A} + p_{2B}x_{2A}, \ldots p_{nB}x_{nA} \]

\[ p_{1A}x_{1A} + p_{2A}x_{2A}, \ldots p_{nA}x_{nA} \]

\[ = w_1 \frac{p_{1B}}{p_{1A}} + w_2 \frac{p_{2B}}{p_{2A}} + \ldots + w_n \frac{p_{nB}}{p_{nA}} \]

where \( w_1 = \frac{p_{1A}x_{1A}}{p_{1A}x_{1A} + p_{2A}x_{2A}, \ldots p_{nA}x_{nA}} \) etc.

The base weighted price index is a weighted average of proportionate price increases where weight for good \( i \) is the proportion of expenditure spent on good \( i \) at date \( A \).
European Union Consumer Price Indices: CPI

Consumer Price Index (CPI)

CPI is essentially base weighted indices but the weights change over time as expenditure patterns change.
Czech republic Consumer Price Indices: CPI

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<th>Category</th>
<th>Euro area (changing composition)</th>
<th>Italy</th>
<th>Slovakia</th>
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European Union weights to different goods.
Gen 2017, Source www.ecb.europa.eu

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Price indices and substitution bias
2. Substitution bias

\[
\frac{p_{1B}x_{1A} + p_{2B}x_{2A}}{p_{1A}x_{1A} + p_{2A}x_{2A}}
\]

The base weighted price index measures the proportional increase in the cost of \((x_{1A}, x_{2A})\)

\[
\frac{E(p_{1B}, p_{2B}, u)}{E(p_{1A}, p_{2A}, u)}
\]

The expenditure function price index measures the proportional increase in the cost of getting utility \(u\)

Fact: base weighted index ≥ expenditure function price index.

The next slides explain why.
\((x_{1A}, x_{2A})\) maximizes utility subject to the budget constraint
\[ p_{1A}x_1 + p_{2A}x_2 = m_1 \]

\((x_{1A}, x_{2A})\) is the cheapest way of getting utility \(u_1\) at prices \((p_{1A}, p_{2A})\) so the expenditure function

\[ E(p_{1A}, p_{2A}, u_1) = p_{1A}x_{1A} + p_{2A}x_{2A} = m_1 \]
\[ m_3 = p_{1B}x_{1A} + p_{2B}x_{2A} \]
is the cost of \((x_{1A}, x_{2A})\) at prices \((p_{1B}, p_{2B})\).

\[ m_1 = p_{1A}x_{1A} + p_{2A}x_{2A} \]
is the cost of \((x_{1A}, x_{2A})\) at prices \((p_{1A}, p_{2A})\).

\[ \frac{m_3}{m_1} = \frac{p_{1B}x_{1A} + p_{2B}x_{2A}}{p_{1A}x_{1A} + p_{2A}x_{2A}} \]

proportional increase in income needed to continue to buy \((x_{1A}, x_{2A})\) after the price change

\[ = \text{ base weighted price index.} \]
\((x_{1B}, x_{2B})\) is the cheapest way of getting utility \(u_1\) at prices \(p_{1B}, p_{2B}\)

so the expenditure function

\[
E(p_{1B}, p_{2B}, u_1) = p_{1B}x_{1B} + p_{2B}x_{2B} = m_2
\]

\((x_{1A}, x_{2A})\) also gives utility \(u_1\) so costs the same or more than \((x_{1B}, x_{2B})\) at prices \(p_{1B}, p_{2B}\)

so \(m_2 = p_{1B}x_{1B} + p_{2B}x_{2B} \leq p_{1B}x_{1A} + p_{2B}x_{2A} = m_3\).
Here $m_2 < m_3$ so a consumer with income $m_3$ has higher utility than a consumer with income $m_2$. 
If prices change from \((p_{1A}, p_{2A})\) to \((p_{1B}, p_{2B})\) and income changes from \(m_1\) to \(m_2\) utility does not change.

If prices change from \((p_{1A}, p_{2A})\) to \((p_{1B}, p_{2B})\) and income changes from \(m_1\) to \(m_3\) utility increases.
Base weighted price index

\[
m_3 = \frac{p_{1B}x_{1A} + p_{2B}x_{2A}}{p_{1A}x_{1A} + p_{2A}x_{2A}}
\]

proportional increase in income needed to continue to buy \((x_{1A}, x_{2A})\) after the price change.

\[\geq ( > \text{if substitution is possible})\]

\[
m_2 = \frac{p_{1B}x_{1B} + p_{2B}x_{2B}}{p_{1A}x_{1A} + p_{2A}x_{2A}}
\]

proportional increase in income needed to continue to have utility \(u_1\) after the price change.

\[
= \frac{E(p_{1B}, p_{2B}, u_1)}{E(p_{1A}, p_{2A}, u_1)}
\]

expenditure function price index
Reminder: the expenditure function is homogeneous of degree 1 in prices

that is if $k > 0$ then

$$E(kp_1, kp_2, u) = kE(p_1, p_2, u)$$

so if $p_{1B} = kp_{1A}$ & $p_{2B} = kp_{2A}$, so both prices grow at the same rate

expenditure function price index

$$\frac{E(p_{1B}, p_{2B}, u_A)}{E(p_{1A}, p_{2A}, u_A)} = \frac{kE(p_{1A}, p_{2A}, u_A)}{E(p_{1A}, p_{2A}, u_A)} = k$$

base weighted price index

$$\frac{p_{1B}x_{1A} + p_{2B}x_{2A}}{p_{1A}x_{1A} + p_{2A}x_{2A}} = \frac{kp_{1A}x_{1A} + kp_{2A}x_{2A}}{p_{1A}x_{1A} + p_{2A}x_{2A}} = k.$$

Both price indices increase at the same rate.
Substitution bias

- If prices do not all grow at the same rate
- & there is a substitution effect
- base weighted price index
  > expenditure function price index.
Substitution bias

If income grows at the same rate as a base weighted price index utility either increases or stays the same.

If there is any possibility of substitution utility increases.

Thus base weighted price indices overstate the rate of inflation.
Over the long term the introduction of new goods is a big issue for price indices.

Who today would buy a film stored on this?

Who in 1990 downloaded a film from the internet?
Problems with the expenditure function price index

- Calculating the expenditure function price index requires knowledge of the expenditure function.

- The expenditure function is derived from the utility function.

- The utility function is unobservable.

- There is a neat way round this for situations where only one price changes.

- This is the compensating and equivalent variation developed by Hicks which turn out to be closely related to consumer surplus.
Compensating variation

CV
The compensating variation for a price increase from $p_{1A}$ to $p_{1B}$ is the amount of extra money the consumer needs to get back to the same level of utility as before the price change.
Compensating variation and the expenditure function

At prices \((p_{1A}, p_2)\) the consumer buys \((x_{1A}, x_{2A})\) giving utility \(u(x_{1A}, x_{2A}) = u_A\).

After the price change & compensation the consumer gets the same level of utility by buying \((x_{1B}, x_{2B})\) so \(u(x_{1B}, x_{2B}) = u_A\).

The consumer minimises the cost of getting utility so

- the amount spent at prices \((p_{1A}, p_2)\) is \(E(p_{1A}, p_2, u_A)\),
- the amount spent at prices \((p_{1B}, p_2)\) after compensation is \(E(p_{1B}, p_2, u_A)\).
- Compensating variation = \(E(p_{1B}, p_2, u_A) - E(p_{1A}, p_2, u_A)\)
Compensating variation

\[ x_2 \]

\[ 0 \leq x_1 \]

\[ A \]
Compensating variation
Compensating variation

A
Compensating variation
Compensating variation
Compensating variation

\[ \text{CV} \]

\[ \frac{p_2}{p_2} \]

\[ E(p_{1B}, p_2, u_A) \]

\[ p_2 \]

\[ x_2 \]

\[ E(p_{1A}, p_2, u_A) \]

\[ p_2 \]

\[ x_1 \]
Compensating variation

\[ \text{CV} = \frac{\text{E}(p_{1B}, p_2, u_A)}{p_2} \]

\[ \text{E}(p_{1A}, p_2, u_A) \]

How much the consumer is available to pay more to maintain the same utility as before price increase.

Diagram:

- Graph showing preferences and budget lines.
- Labeled points and vectors indicating compensating variation.
- Explanation: The compensating variation (CV) is the change in income that makes the consumer indifferent to the price increase, effectively compensating for the loss in utility.
Equivalent variation
EV
4. Equivalent Variation (EV)

Definition

EV is the amount of money that taken away from the consumer without changing prices has the same effect on utility as the price change.

\[ EV = E(p_{1B}, p_2, u_B) - E(p_{1A}, p_2, u_B) \]

The next slide shows why.
Equivalent variation
Equivalent variation
Equivalent variation
Equivalent variation
Equivalent variation
Equivalent variation
Equivalent variation
Equivalent variation

\[ EV(p_{1B}, p_2, u_B) = E(p_{1A}, p_2, u_B) \]
Equivalent variation

How much the consumer is available to income less in order to avoid price increase

\[ E(p_{1B}, p_2, u_B) \]

\[ E(p_{1A}, p_2, u_B) \]
5. Compensated variation vs Equivalent variation

\[
\text{CV} = E(p_{1B}, p_2, u_A) - E(p_{1A}, p_2, u_A) \\
\text{EV} = E(p_{1B}, p_2, u_B) - E(p_{1A}, p_2, u_B)
\]
Compensated variation vs Equivalent variation

Compensated variation = we measure it on the compensated demand on initial utility indifference curve. When price changes and we want to keep same utility, how much are we willing to pay?

Equivalent variation = we measure it on the compensated demand on new utility indifference curve. When price changes how much are we willing to pay to go back to previous utility?

\[ CV = E(p_{1B}, p_2, u_A) - E(p_{1A}, p_2, u_A) \]

\[ EV = E(p_{1B}, p_2, u_B) - E(p_{1A}, p_2, u_B) \]
Compensated variation vs Equivalent variation

Compensated variation =
price changes $\rightarrow$
how much does it cost to keep same utility?

$$CV = E(p_{1B}, p_2, u_A) - E(p_{1A}, p_2, u_A)$$

Equivalent variation =
price changes $\rightarrow$
utility reduces $\rightarrow$
how much would we save to have the same new utility if the prices would not change?

$$EV = E(p_{1B}, p_2, u_B) - E(p_{1A}, p_2, u_B)$$
Is it possible to measure CV and EV with compensated and uncompensated demand function?
CV and Shephard’s Lemma

• The problem remains, CV depends on the expenditure function so depends on utility so it is unobservable.

• Remember Shephard’s lemma

\[
\frac{\partial E(p_1, p_2, u)}{\partial p_1} = h_1(p_1, p_2, u)
\]

• To work with this you need to remember the relationship between differentiation and integration.
The relationship between differentiation and integration

If \( y = f(x) \)

\[
\frac{dy}{dx} = f'(x)
\]

then \( f(b) - f(a) = \int_{a}^{b} f'(x) \, dx \)
$\int_{p_1A}^{p_1B} h_1(p_1, p_2, u_A) dp_1$

= compensating variation when $p_1$ rises from $p_{1A}$ to $p_{1B}$.

compensated demand curve $h_1(p_1, p_2, u_A)$

Price increases from $p_{1A}$ to $p_{1B}$

compensated demand falls from $x_{1A}$ to $x_{1B}$
CV and Shephard’s Lemma

Shephard’s lemma

\[ \frac{\partial E(p_1, p_2, u)}{\partial p_1} = h_1(p_1, p_2, u) \]

implies that

\[ \int_{p_{1A}}^{p_{1B}} h_1(p_1, p_2, u) dp_1 = E(p_{1B}, p_2, u) - E(p_{1A}, p_2, u) \]
$u_A$ is the level of utility that the consumer gets with prices $(p_{1A}, p_2)$ and income $m$.

Demand curve diagram

- Compensated demand: $h_1(p_1, p_2, u_A)$
- Uncompensated demand: $x_1(p_1, p_2, m)$

Demand curve diagram
Compensated demand is less elastic than uncompensated demand. Income and substitution effects work in the same direction. This is a **normal** good.

**Demand curve diagram**
Demand curve diagram

compensated demand
\( h_1(p_1, p_2, u_A) \)

uncompensated demand
\( x_1(p_1, p_2, m) \)
$h_1(p_1, p_2, u_B)$

$u_B$ is the level of utility that the consumer gets with prices $(p_{1B}, p_2)$ and income $m$.

$u_A$ is the level of utility that the consumer gets with prices $(p_{1A}, p_2)$ and income $m$.

Demand curve diagram
compensated demand
\[ h_1(p_1,p_2,u_B) \]

Demand curve diagram

Compensating Variation ACDF

\[ h_1(p_1,p_2,u_A) \]
compensated demand
$h_1(p_1, p_2, u_B)$

uncompensated demand
$x_1(p_1, p_2, m)$

Demand curve diagram
Normal good
EV < CV
ABEF ACDF

Demand curve diagram
Normal goods, CV & EV

Uncompensated demand is more elastic than compensated demand because income and substitution effects work in the same direction.

For a price rise

EV < CV

because EV is measured at a lower level of utility.

As the good is normal, it is less consumed at lower utility.
Compensating variation, equivalent variation and change in consumer surplus
6. Compensating variation, equivalent variation and change in consumer surplus

Demand curve diagram

Compensating Variation ACDF

Compensated demand

\[ h_1(p_1, p_2, u_A) \]
compensated demand
\[ h_1(p_1,p_2,u_B) \]

uncompensated demand
\[ x_1(p_1,p_2,m) \]

Demand curve diagram
uncompensated demand $x_1(p_1,p_2,m)$

Demand curve diagram
compensated demand
\[ h_1(p_1, p_2, u_B) \]

Normal good
EV < change CS < CV
ABEF ABDF ACDF

Demand curve diagram

uncompensated demand
\[ x_1(p_1, p_2, m) \]
Normal goods, CV, consumer surplus & EV

Change in Consumer Surplus is the area bounded by the \textit{uncompensated} demand curve.

Compensating variation is the area bounded by the \textit{compensated} demand curve with utility $u_A$.

Equivalent variation is the area bounded by the \textit{compensated} demand curve with utility $u_B$.

For a price rise

\[ EV < \text{change in consumer surplus} < CV \]
Income effects, CV, EV and consumer surplus
7. Income effects, CV, EV and consumer surplus

When there are no income effects uncompensated and compensated demand are the same so the loss in consumer surplus due to an increase in $p_1$ is the same as the CV & EV.

The difference between CV, EV and the change in consumer surplus is due to income effect.
The Slutsky equation in elasticities shows the size of the income effect

\[
\frac{p_1}{x_1} \frac{\partial x_1}{\partial p_1} = \frac{p_1}{h_1} \frac{\partial h_1}{\partial p_1} - \frac{m}{x_1} \frac{\partial x_1}{\partial m} \frac{p_1 x_1}{m}
\]
Income effects are small when either or both of the income elasticity of uncompensated demand and the budget share are small.

If income effects are small the change in consumer surplus is a good approximation to the compensating variation.
Which measure to use?

If income effects are small, for example because the budget share is small, CV, EV and change in CS are close. Use change in CS because it is easy to measure.

If income effects are large, it depends on the question you want to answer.

CV if it is how much is needed to compensate.

EV if it is what is the monetary equivalent of the price change.
Examples

A government wants to reduce CO$_2$ generation to combat global warming.

Fuel is a large share of expenditure so the income effect is significant.

The gov does this through taxation.

To make this politically acceptable the gov needs to compensate people for the increase in tax, the compensating variation is relevant. For instance increasing public investment in green areas.

Otherwise if the gov wants to make some tax reduction to balance, he has to measure the monetary effect with the equivalent variation.
When does an increase in price from $p_{1A}$ to $p_{1B}$ have a big adverse impact on the consumer?
Fall in consumer surplus due to $p_1$ increasing from $p_{1A}$ to $p_{1B}$. 

In which diagram is demand more elastic A or B? In which diagram is the fall in consumer surplus bigger, A or B?
In which diagram is demand more elastic A or B?

In which diagram is the fall in consumer surplus bigger, A or B?

Fall in consumer surplus due to $p_1$ increasing from $p_{1A}$ to $p_{1B}$.

Diagram A

Diagram B

demand curve diagram
Fall in consumer surplus due to $p_1$ increasing from $p_{1A}$ to $p_{1B}$.

In which diagram is demand more elastic A or B?

In which diagram is the fall in consumer surplus bigger, A or B?
The fall in consumer surplus due to a price increase from $p_{1A}$ to $p_{1B}$ is less than $(p_{1B} - p_{1A})x_{1A}$ the extra money needed to buy $x_{1A}$.

The fall in consumer surplus is less when demand is elastic, usually because there is a good substitute available.
Formula for fall into consumer surplus.

Let $\Delta p_1 = p_{1B} - p_{1A} > 0$

Let $\Delta x_1 = x_{1B} - x_{1A} < 0$

\[
\text{fall in consumer surplus} = (p_{1B} - p_{1A})x_{1A} - \frac{1}{2}(p_{1B} - p_{1A})(x_{1A} - x_{1B})
\]

\[
= \Delta p_1 x_{1A} + \frac{1}{2} \Delta p_1 \Delta x_1
\]
Fall in Consumer Surplus

$$\Delta p_1 x_{1A} + \frac{1}{2} \Delta p_1 \Delta x_1$$

$$= \Delta p_1 x_{1A} \left( 1 + \frac{1}{2} \frac{\Delta x_1}{x_{1A}} \right) = \Delta p_1 x_{1A} \left( 1 + \frac{1}{2} \left( \frac{\Delta x_1}{\Delta p_1} \frac{p_1}{x_{1A}} \right) \frac{\Delta p_1}{p_1} \right)$$

$$= \Delta p_1 x_{1A} \left( 1 + \frac{1}{2} e \frac{\Delta p_1}{p_1} \right) \text{ where } e = \frac{\Delta x_1}{\Delta p_1} \frac{p_1}{x_{1A}} < 0 = \text{elasticity}$$
The fall in consumer surplus is less when demand is more elastic.

Fall in consumer surplus = $\Delta p_1 x_{1A} \left(1 + \frac{1}{2} e \frac{\Delta p_1}{p_{1A}} \right)$

where

$\Delta p_1 x_{1A}$ is the increase in the cost of buying $x_{1A}$

$e = \frac{\Delta x_1}{\Delta p_1} \frac{p_{1A}}{x_{1A}}$ own price elasticity of demand $\leq 0$

$\frac{\Delta p_1}{p_{1A}}$ proportionate price increase

The higher the elasticity, since it is negative, the less the fall in consumer surplus
Why is the fall in consumer surplus for a given price change less when demand is more elastic?

Demand is elastic when there is a good substitute available.

Then is not so painful to switch between one good to the other.
When does an increase in price from $p_{1A}$ to $p_{1B}$ have a big adverse impact on the consumer?

- Inelastic demand
- Steep demand curve
- No close substitute available
Adding up demand curves and consumer surplus

Add the demand curves

price

quantity
Adding up demand curves and consumer surplus

Add the demand curves **horizontally**
Adding up demand curves and consumer surplus

Add the demand curves

Shaded area = loss of consumer surplus from group 1 + loss of consumer surplus from group 2
Each person in group 1 earns €8,000 per year.

Each person in group 2 earns €100,000 per year.

There are 100 people in each group.

Situation 1, everyone in group 1 losses CS €250, everyone in group 2 looses CS €50. Total loss CS

= ?

Situation 2, everyone in group 1 losses CS €50, everyone in group 2 looses CS €250. Total loss CS

= ?
Each person in group 1 earns €8,000 per year.

Each person in group 2 earns €100,000 per year,

There are 100 people in each group.

Situation 1, everyone in group 1 losses CS €250, everyone in group 2 looses CS €50. Total loss CS

\[= 100 \times €250 + 100 \times €50 = €30000.\]

Situation 2, everyone in group 1 losses CS €50, everyone in group 2 looses CS €250. Total loss CS

\[= \text{?}\]
Each person in group 1 earns €8,000 per year.

Each person in group 2 earns €100,000 per year,

There are 100 people in each group.

Situation 1, everyone in group 1 losses CS €250, everyone in group 2 looses CS €50. Total loss CS

= 100 x €250 + 100 x €50= €30000.

Situation 2, everyone in group 1 losses CS €50, everyone in group 2 looses CS €250. Total loss CS

= 100 x €50 + 100 x €250= €30000.
Adding up consumer surplus geometrically implies a value judgement that giving €1 to one consumer has the same social benefit as giving €1 to any other consumer.

If you disagree with this judgement you would want to evaluate the losses to each group, and then consider how to use them as input into a decision.

Very important

This can be modelled mathematically.
Using equivalent variation to assess the effect of a tax
8. Using EV to assess the effect of a tax

Suppose that a tax causes the price of good 1 to rise from $p_{1A}$ to $p_{1A} + t$, where $t$ is tax. (This is called an excise tax.)

(We will see later that this is a special case, it happens when supply is perfectly elastic.)

Demand for good 1 falls from $x_{1A}$ to $x_{1B}$. Demand for good 2 rises from $x_{2A}$ to $x_{2B}$.

How much revenue does the tax raise?
Tax revenue \( t \times x_{1B} \)

Good 2 is expenditure on other goods, assume \( p_2 = 1 \).

From the budget constraint without tax

\[ p_{1A} x_{1A} + x_{2A} = m \]

budget constraint with tax

\[ (p_{1A} + t) x_{1B} + x_{2B} = m \]

Subtract these equations to get

\[ (p_{1A} x_{1A} + x_{2A}) - (p_{1A} x_{1B} + x_{2B}) - t x_{1B} = 0. \]
Subtract these equations to get

\[(p_{1A} x_{1A} + x_{2A}) - (p_{1A} x_{1B} + x_{2B}) - t x_{1B} = 0.\]

So tax revenue = \( t x_{1B} = (p_{1A} x_{1A} + x_{2A}) - (p_{1A} x_{1B} + x_{2B}) \)

Cost of original combination \((x_{1A}, x_{2A})\) at prices \(p_{1A}, p_2\)

Cost of new combination \((x_{1B}, x_{2B})\) at prices \(p_{1A}, p_2\)
Budget line with tax,
new prices $\rightarrow$ slope $- (p_{1A} + t)$

Budget lines without tax,
original prices $\rightarrow$ slope $- p_{1A}$

Tax revenue

Excess burden =
EV - tax revenue

IC diagram, $p_2 = 1$
IC diagram, \( p_2 = 1 \)

- Tax revenue: \( AB \)
- Excess burden: \( EV - \text{tax revenue} \)

Budget lines without tax, original prices → slope: \(-p_{1A}\)

Budget line with tax, new prices → slope: \(-\left(p_{1A} + t\right)\)
Tax revenue AB

Excess burden = EV - tax revenue

IC diagram, p₂ = 1
Tax revenue $AB$

Excess burden =

$EV - \text{tax revenue}$

$EV$ $AC$

$\text{budget line with tax, new prices} \rightarrow \text{slope} - (p_{1A} + t)$

$\text{budget lines without tax, original prices} \rightarrow \text{slope} - p_{1A}$

IC diagram, $p_2 = 1$
Excess burden = EV – tax revenue

Budget lines with tax:
- New prices: slope = $- (p_{1A} + t)$
- Original prices: slope = $- p_{1A}$

Budget line with tax:
- New prices: $x_1 = \frac{\text{EV} - \text{tax revenue}}{p_{1A} + t}$
- Original prices: $x_1 = \frac{\text{EV} - \text{tax revenue}}{p_{1A}}$

Budget lines without tax:
- New prices: $x_1 = \frac{\text{EV} - \text{tax revenue}}{p_{1A}}$
- Original prices: $x_1 = \frac{\text{EV} - \text{tax revenue}}{p_{1A}}$

IC diagram, $p_2 = 1$
IC diagram, $p_2 = 1$

**Budget line with tax**

$$\text{slope} = - (p_{1A} + t)$$

**Budget lines without tax**

$$\text{slope} = - p_{1A}$$

**Excess burden**

$$\text{Excess burden} = \text{EV} - \text{tax revenue} = \text{BC}$$

**Tax revenue**

$$\text{AB}$$

**EV**

$$\text{AC}$$

**IC diagram**

$$\text{A} \rightarrow \text{B} \rightarrow \text{C}$$
Definition: The excess burden of an excise tax is

\[ EV - \text{tax revenue} = \text{monetary loss to consumer} - \text{tax revenue} \]

The total society benefit decreases by EV and increases by the tax revenue, but in total it decreases by the excess burden.

With an excise tax there is an excess burden.

Suppose that instead of a excise tax \( t \), the government imposed a “lump sum” tax that took away \( R \) from the consumer so the budget constraint is

\[ p_1 A x_1 + p_2 x_2 = m - R. \]

With \( R = t x_{1B} \)
The lump sum tax raises the same revenue as the excise tax but gives higher utility.
This is a general argument.

Suppose the government wants to raise revenue $R$.

A **lump sum tax** that reduces income by $R$ that does not depend on anything the consumer does reduces utility by less than a tax raises $R$ where the revenue could be changed by changing consumption, work or saving.

(e.g. excise tax, VAT, income tax…)

The only feasible lump sum tax is a “poll tax” where everyone pays the same amount.

Is a poll tax ethically desirable?

Is a poll tax politically feasible?
Using equivalent variation to assess the effect of a subsidy.

Suppose that a subsidy causes the price of good 1 to fall from $p_{1A}$ to $p_{1A} - s$, where $s$ is the subsidy per unit.

Demand for good 1 changes from $x_{1A}$ to $x_{1C}$. Demand for good 2 changes from $x_{2A}$ to $x_{2C}$.

How much does the subsidy cost?
IC diagram, $p_2 = 1$

- Budget line with subsidy:
  - Slope: $(p_{1A} - s)$
  - Point: $(x_{1C}, x_{2C})$

- Budget line without subsidy:
  - Slope: $p_{1A}$
  - Point: $(x_{1A}, x_{2A})$
Subsidy costs $s \times x_{1C}$

Good 2 is expenditure on other goods so $p_2 = 1$.

From the budget constraint without subsidy

$$p_{1A} x_{1A} + x_{2A} = m$$

budget constraint with subsidy

$$(p_{1A} - s) x_{1C} + x_{2C} = m$$

Subtract these equations to get

$$(p_{1A} x_{1A} + x_{2A}) - (p_{1A} x_{1C} + x_{2C}) + s x_{1C} = 0.$$
Subtract these equations to get

\[(p_{1A} x_{1A} + x_{2A}) - (p_{1A} x_{1C} + x_{2C}) + s x_{1C} = 0.\]

Rearrange cost of subsidy

\[= s x_{1C} = (p_{1A} x_{1C} + x_{2C}) - (p_{1A} x_{1A} + x_{2A})\]

Cost of new bundle 
\[(x_{1C},x_{2C}) \text{ at prices } p_{1A}, p_{2} = 1.\]

Cost of original bundle 
\[(x_{1A},x_{2A}) \text{ at prices } p_{1A}, p_{2} = 1.\]
IC diagram, $p_2 = 1$

Cost of subsidy?

_budget line with subsidy gradient – $(p_{1A} - s)$_

_budget lines gradient – $p_{1A}$_

$(x_{1A}, x_{2A})$

$(x_{1C}, x_{2C})$
Cost of subsidy? AC

(\(x_{1A}, x_{2A}\))

(budget lines gradient \(- (p_{1A} - s)\))

(budget line with subsidy)

IC diagram, \(p_2 = 1\)
The equivalent variation (EV) of a subsidy is the amount of extra income the consumer needs to get to have the same effect on utility as the subsidy.
IC diagram, $p_2 = 1$

- **Cost of subsidy?**

- **Equivalent variation?**

- **Budget line with subsidy gradient** – $(p_{1A} - s)$

- **Budget lines gradient** – $p_{1A}$

- $(x_{1A}, x_{2A})$

- $(x_{1C}, x_{2C})$
Equivalent variation? BC

budget line with subsidy gradient – \( (p_{1A} - s) \)

budget lines gradient – \( p_{1A} \)

IC diagram, \( p_2 = 1 \)
Cost of subsidy? AC
Equivalent variation? BC

budget line with subsidy gradient – (p_{1A} – s)
budget lines gradient – p_{1A}

IC diagram, p_2 = 1
Cost of subsidy? AC > equivalent variation BC

budget line with subsidy gradient – (p_{1A} - s)
budget line without subsidy gradient – p_{1A}

IC diagram, p_2 = 1
General argument

- A lump sum subsidy increases income by a fixed amount that does not depend on anything the consumer does.

- Increasing the consumer’s utility by giving the EV as a lump sum costs less than increasing the consumer’s utility by the same amount using a subsidy.

- So either the state can save some money and get the same EV, or can use the same amount of money and get a higher EV (higher utility) as in next slide.
The diagram illustrates an indifference curve (IC) diagram with a budget line and budget lines with and without subsidy gradients. The variables are labeled as follows:

- \( p_2 = 1 \)
- \( x_1 \)
- \( x_2 \)

The budget line without subsidy gradient is given by:

\[
\text{budget line without subsidy gradient} = p_1A - p_1A - s
\]

The budget line with subsidy gradient is given by:

\[
\text{budget line with subsidy gradient} = (p_1A - s)
\]

The cost of subsidy is indicated as AC, and the equivalent variation is BC. The diagram shows the movement from point A to B to C, illustrating the impact of subsidy on consumer choice.
Does compensating a consumer for a price increase imply that the price increase has no effect on demand?
10. Does compensating a consumer for a price increase imply that the price increase has no effect on demand?

Suppose a price (e.g. heating fuel) rises from $p_{1A}$ to $p_{1B}$. Demand for the good falls.

Suppose the consumer is compensated by being given the compensating variation (CV). Does the consumer go back to consuming the same amount of heating fuel?
The price increase moves the consumer from $\text{A}$.

Following compensation the consumer is at $\text{B}$.

Is the substitution effect of the price rise. Does it increase or decrease $x_1$? $\text{CV}$?

IC diagram, $p_2 = 1$
The price increase moves the consumer from A to C. Following compensation the consumer is at B. Is the substitution effect of the price rise. Does it increase or decrease $x_1$? CV?
The price increase moves the consumer from A to C. Following compensation, the consumer is at B. Is the substitution effect of the price rise. Does it increase or decrease $x_1$? CV?
The price increase moves the consumer from A to C

Following compensation the consumer is at B

A to B is the substitution effect of the price rise. Does it increase or decrease $x_1$? CV?
The price increase moves the consumer from A to C.

Following compensation the consumer is at B.

A to B is the substitution effect of the price rise. Does it increase or decrease $x_1$? CV?
The price increase moves the consumer from A to C

Following compensation the consumer is at B

A to B is the substitution effect of the price rise. Does it increase or decrease $x_1$? CV? DE
• A price increase reduces demand even if the consumer is compensated.

• This is an economists’ insight.

• It comes from knowing about income and substitution effects.

• If income effects are small compensation has little effect on the demand for the good.
Have you received a present and thought you would rather have the money it cost?

1. often
2. rarely
3. never
Benefits in kind
The consumer gets a benefit in kind, getting $x_{1D}$ units of good 1 costing $p_1 x_{1D}$ for free.
In this figure would the consumer be better off just getting the money as extra income?
In this figure would the consumer be better off just getting the money as extra income? No
In this figure would the consumer be better off just getting the money as extra income?
In this figure would the consumer be better off just getting the money as extra income?
Yes

IC diagram, $p_2 = 1$
Why Benefits in Kind?

The last slides suggest that it is sometimes better and never worse for a consumer to get a sum of money rather than a benefit in kind costing the same amount.

So why are benefits in kind common?
Why Benefits in Kind?

Screen Actors Guild Gift Bags © Getty Images
Why do we give gifts not money?

Mother and daughter (18-21 months) giving food basket to senior woman © Getty Images
Why do governments provide health and education free at the point of service?
Price changes and welfare.
What have we achieved?

• Understanding of uses and limitations of price indices.

• Monetary measures of impact of price changes on consumer’s welfare, & the implicit value judgement.

• Application of these measures to taxes & subsidies
  • Change in consumer surplus, compensating & equivalent variation.

• Insights on effect of compensation on consumer demand.