I. SERIE

① $\frac{x+4}{x-2} \geq \frac{x+1}{x+3}$; $x \neq -3, 2$; a) $x>2$: $(x+4)(x+3) \geq (x-2)(x+1)$

$x^2+7x+12 \geq x^2-x-2 \Leftrightarrow x \geq -\frac{7}{4}$: $(2,+\infty)$

b) $x \in (-3,2)$: viz a) $\rightarrow x \leq -\frac{7}{4}$: $(-3,-\frac{7}{4}\rangle$

c) $x<-3$: viz a) $\rightarrow x \geq -\frac{7}{4}$: $\emptyset$

Výsledek: $(-3,-\frac{7}{4}\rangle \cup (2,+\infty)$

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② $|x|x-3ax+5>0$, $a \in \mathbb{R}$

i) $x \geq 0$: $x^2-3ax+5>0$; $x_{1,2}=\frac{3a \pm \sqrt{9a^2-20}}{2}$

I) $9a^2-20<0 \Leftrightarrow a \in (-\frac{\sqrt{20}}{3},\frac{\sqrt{20}}{3})$: $x \in \mathbb{R} \wedge \mathbb{R}^+$ $\Rightarrow x \in \langle 0,+\infty)$

II) $a \notin (-\frac{\sqrt{20}}{3},\frac{\sqrt{20}}{3})$: $x \in \langle 0,+\infty) \cap \left((-\infty,\frac{3a-\sqrt{9a^2-20}}{2}) \cup (\frac{3a+\sqrt{9a^2-20}}{2},+\infty)\right) = (*)$

$a \leq -\frac{\sqrt{20}}{3}$: $(*) = \langle 0,+\infty)$

$a \geq \frac{\sqrt{20}}{3}$: $(*) = \langle 0,\frac{3a-\sqrt{9a^2-20}}{2}) \cup (\frac{3a+\sqrt{9a^2-20}}{2},+\infty)$

ii) $x<0$: $-x^2-3ax+5>0 \Leftrightarrow x^2+3ax-5<0$; $x_{1,2}=\frac{-3a \pm \sqrt{9a^2+20}}{2}$

$x \in (\frac{-3a-\sqrt{9a^2+20}}{2}, 0)$

Výsledek: $a \leq -\frac{\sqrt{20}}{3}$: $(\frac{-3a-\sqrt{9a^2+20}}{2},+\infty)$

$a \in (-\frac{\sqrt{20}}{3},\frac{\sqrt{20}}{3})$: — // —

$a \geq \frac{\sqrt{20}}{3}$: $(\frac{-3a-\sqrt{9a^2+20}}{2},\frac{3a-\sqrt{9a^2-20}}{2}) \cup (\frac{3a+\sqrt{9a^2-20}}{2},+\infty)$

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③ $2\log_{\frac{1}{2}}(|x|-1) \leq \log_{\frac{1}{2}}(x^2-7x+12)$

A) $|x|-1>0 \Leftrightarrow x \notin \langle -1,1\rangle$

B) $x^2-7x+12>0$; $x_{1,2}=\frac{7\pm\sqrt{49-48}}{2}=\begin{cases}4\\3\end{cases}$: $x \notin \langle 3,4\rangle$

c) $\log_{\frac{1}{2}}$ je $\downarrow$, $2\log y = \log y^2$ pro $y>0$: $(|x|-1)^2 \geq x^2-7x+12$

$(|x|-1)^2 = x^2-2|x|+1$

$\Leftrightarrow 7x-2|x| \geq 11$

i) $x \geq 0$: $5x \geq 11 \Leftrightarrow x \geq \frac{11}{5}$

ii) $x<0$: $9x \geq 11 \Leftrightarrow x \geq \frac{11}{9}$ nelze

$\Rightarrow x \geq \frac{11}{5}$

Dohromady: $x \in \langle \frac{11}{5},3) \cup (4,+\infty)$

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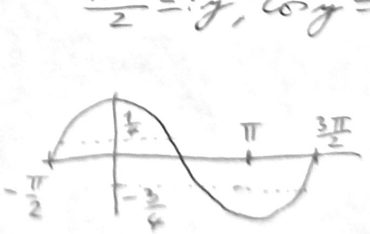
④ $13\cos^2(\frac{x+3}{2})+8\cos(\frac{x+3}{2})-3\sin^2(\frac{x+3}{2})>0$ $\frac{x+3}{2}=:y$; $\cos y = t$

$13\cos^2(\frac{x+3}{2})+8\cos(\frac{x+3}{2})-3\sin^2(\frac{x+3}{2}) = 16t^2+8t-3$

$16t^2+8t-3>0$; $t_{1,2}=\frac{-8\pm\sqrt{64+12\cdot 16}}{32}=\begin{cases}\frac{1}{4}\\-\frac{3}{4}\end{cases}$

$\cos y > \frac{1}{4} \vee \cos y < -\frac{3}{4}$



a) $y \in \langle 0,\pi\rangle$: $\cos y = \frac{1}{4} \Leftrightarrow y = \arccos\frac{1}{4}$

$\cos y = -\frac{3}{4} \Leftrightarrow y = \arccos(-\frac{3}{4})$

b) $y \in \langle -\frac{\pi}{2},\frac{3\pi}{2}\rangle$: $\cos y = \frac{1}{4} \Leftrightarrow y = \pm\arccos(\frac{1}{4})$

$\cos y = -\frac{3}{4} \Leftrightarrow y = \arccos(-\frac{3}{4}) \vee 2\pi - \arccos(-\frac{3}{4})$

$\cos y > \frac{1}{4} \Leftrightarrow y \in (-\arccos(\frac{1}{4}),\arccos(\frac{1}{4})) =: M$

$\cos y < -\frac{3}{4} \Leftrightarrow y \in (\arccos(-\frac{3}{4}), 2\pi-\arccos(-\frac{3}{4})) =: N$

c) $y \in \mathbb{R} : y \in \bigcup_{k \in \mathbb{Z}} \left((M \cup N) + 2k\pi \right)$

$$\frac{x+3}{2} = y : x \in \bigcup_{k \in \mathbb{Z}} \left((2M \cup 2N) + 4k\pi - 3 \right) = \bigcup_{k \in \mathbb{Z}} \Big(\big(-2\arccos(\tfrac{1}{4}) + 4k\pi - 3, \overbrace{2\arccos(\tfrac{1}{4})}^{+4k\pi-3}\big) \cup$$

$$\cup \big(2\arccos(-\tfrac{3}{4}) + 4k\pi - 3, -2\arccos(-\tfrac{3}{4}) + 4(k+1)\pi - 3\big)\Big).$$

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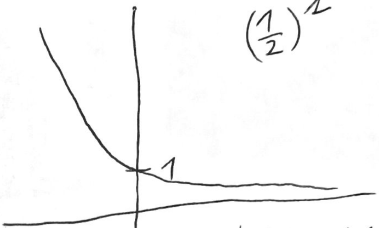
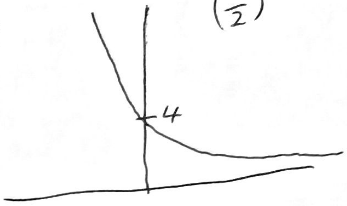
⑤ $f(x) = \left| \left(\tfrac{1}{2}\right)^{||x|-1|-2} - 1 \right|$:

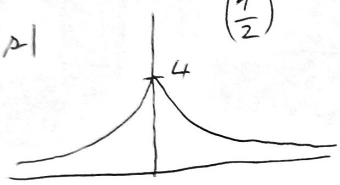
$\left(\frac{1}{2}\right)^{t}$ $\quad t = w - 2$



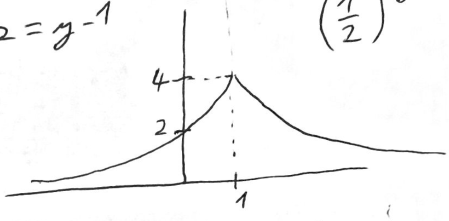
$\left(\frac{1}{2}\right)^{w-2}$



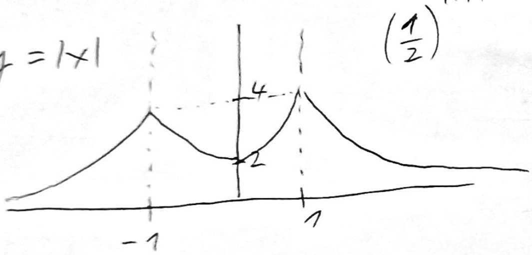
$w = |r|$ $\quad \left(\frac{1}{2}\right)^{|r|-2}$



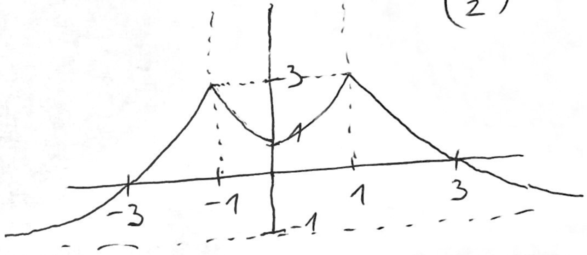
$r = y - 1$ $\quad \left(\frac{1}{2}\right)^{|y-1|-2}$



$y = |x|$ $\quad \left(\frac{1}{2}\right)^{||x|-1|-2}$



$\left(\frac{1}{2}\right)^{||x|-1|-2} - 1$



$\left| \left(\frac{1}{2}\right)^{||x|-1|-2} - 1 \right|$