

II. SERIE

① $4^x - 3 \cdot 2^{x+1} + 8 \geq 0; 2^x = t > 0$
 $t^2 - 6t + 8 \geq 0; t_{1,2} = \frac{6 \pm \sqrt{36-32}}{2} = \begin{cases} 4 \\ 2 \end{cases} \quad t^1=2; t^2=4$
 $t \in (0, 2) \cup (4, +\infty) \Rightarrow x \in (-\infty, 1) \cup (2, +\infty)$

② $|x+1| + |2x-3| = |2x-1| + 4$; Dial. body: $\{-1, \frac{1}{2}, \frac{3}{2}\}$

a) $x \geq \frac{3}{2}$: $x+1+2x-3 = 2x-1+4$

$3x-2 = 2x+3 \Rightarrow x = 5 \geq \frac{3}{2} \checkmark$

Wy'sledok: $x \in \{-3, 5\}$

b) $x \in (\frac{1}{2}, \frac{3}{2})$: $x+1-2x+3 = 2x-1+4$

$-x+4 = 2x+3 \Rightarrow x = \frac{1}{3} < \frac{1}{2} \times$

c) $x \in (-1, \frac{1}{2})$: $x+1-2x+3 = -2x+1+4$

$-x+4 = -2x+5 \Rightarrow x = 1 > \frac{1}{2} \times$

d) $x < -1$: $-x-1-2x+3 = -2x+1+4$

$-3x+2 = -2x+5 \Rightarrow x = -3 < -1 \checkmark$

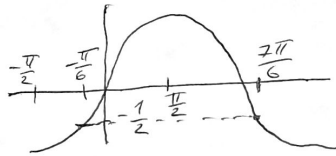
③ $2 \sin^3(2x-1) - \sin^2(2x-1) - \sin(2x-1) = 0; 2x-1 = y; \sin y = t$

$2t^3 - t^2 - t = 0 \Rightarrow t=0 \vee 2t^2 - t - 1 = 0; t_{1,2} = \frac{1 \pm \sqrt{1+8}}{4} = \begin{cases} 1 \\ -\frac{1}{2} \end{cases} \quad t \in \{0, 1, -\frac{1}{2}\}$

$y \in \{k\pi, \frac{\pi}{2} + 2k\pi, -\frac{\pi}{6} + 2k\pi, \frac{7\pi}{6} + 2k\pi; k \in \mathbb{Z}\}$
 $\sin y = 0 \Leftrightarrow y = k\pi, k \in \mathbb{Z}$

$\sin y = 1 \Leftrightarrow y = 2k\pi + \frac{\pi}{2}, k \in \mathbb{Z}$

$\sin y = -\frac{1}{2} \Leftrightarrow y = 2k\pi + \frac{7\pi}{6} \vee y = 2k\pi + \frac{\pi}{6}, k \in \mathbb{Z}$



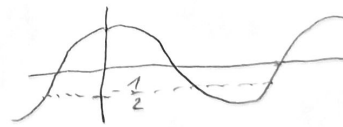
$y = 2x-1 \Rightarrow x \in \left\{ \frac{k\pi+1}{2}, \frac{\pi+2}{4} + k\pi, \frac{6-\pi}{12} + k\pi, \frac{6+7\pi}{12} + k\pi; k \in \mathbb{Z} \right\}$

④ $2 \sin^2(|x|-1) + 3 \cos(|x|-1) = 0 \Leftrightarrow 2 \cos^2 y - 3 \cos y - 2 = 0$

$2t^2 - 3t - 2 = 0; t_{1,2} = \frac{3 \pm \sqrt{9+16}}{4} = \begin{cases} 2 \\ -\frac{1}{2} \end{cases}$

$\cos y \neq 2; \cos y = -\frac{1}{2} \Leftrightarrow$

$y = \pm \frac{2}{3}\pi + 2k\pi, k \in \mathbb{Z}$



$|x|-1 = \pm \frac{2}{3}\pi + 2k\pi \Leftrightarrow |x| = \pm \frac{2}{3}\pi + 1 + 2k\pi$

a) $|x| = \frac{2}{3}\pi + 1 + 2k\pi \Rightarrow k \in \mathbb{N}_0, x = \pm \left(\frac{2}{3}\pi + 1 + 2k\pi \right)$

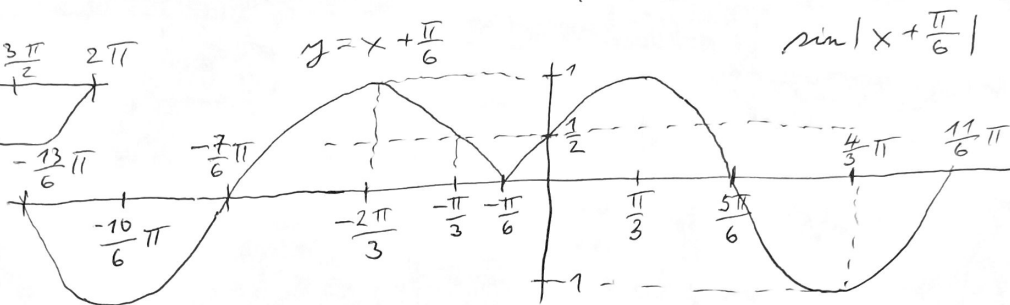
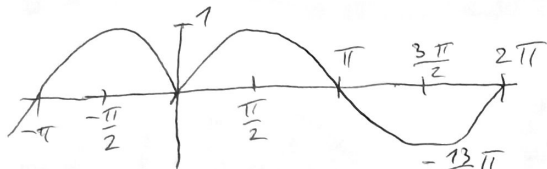
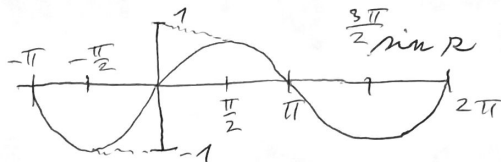
b) $|x| = -\frac{2}{3}\pi + 1 + 2k\pi \Rightarrow k \in \mathbb{N}, x = \pm \left(-\frac{2}{3}\pi + 1 + 2k\pi \right)$

Wy'sledok: $x \in \left\{ \pm \left(\frac{2}{3}\pi + 1 + 2k\pi \right); k \in \mathbb{N}_0 \right\} \cup \left\{ \pm \left(-\frac{2}{3}\pi + 1 + 2k\pi \right); k \in \mathbb{N} \right\}$

$$⑤ f(x) := \left| \sin \left| x + \frac{\pi}{6} \right| - \frac{1}{2} \right|$$

$$r = |y|$$

$$\sin |y|$$



$f(x)$:

