III. SÉRIE

① $\dfrac{|x|-2}{x+3} = \dfrac{x+1}{x-2}$; $x \notin \{-3, 2\}$

a) $x \geq 0;\ x \neq 2:\ (x-2)^2 = (x+3)(x+1) \Leftrightarrow x^2 - 4x + 4 = x^2 + 4x + 3 \Leftrightarrow 8x = 1 \Leftrightarrow \underline{x = \frac{1}{8}}$

$\frac{1}{8} \neq 2,\ \frac{1}{8} \geq 0$ ✓

b) $x < 0;\ x \neq -3:\ (-x-2)(x-2) = x^2 + 4x + 3 \Leftrightarrow -x^2 + 4 = x^2 + 4x + 3$

$\Leftrightarrow 2x^2 + 4x - 1 = 0;\ x_{1,2} = \dfrac{-4 \pm \sqrt{16+8}}{4} = \dfrac{-2 \pm \sqrt{6}}{2} \neq -3$ $\quad \underline{\dfrac{-2-\sqrt{6}}{2}} < 0$ ✓

$\dfrac{-2+\sqrt{6}}{2} \not< 0$ ×

Výsledek: $\underline{x \in \left\{-1 - \frac{\sqrt{6}}{2}, \frac{1}{8}\right\}}$

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② $3\left(\frac{1}{9}\right)^{t^2} = \frac{1}{9}(27)^t \Leftrightarrow 3^{-2t^2+1} = 3^{3t-2} \Leftrightarrow -2t^2 + 1 = 3t - 2 \Leftrightarrow$

3^y je prostá

$2t^2 + 3t - 3 = 0;\ t_{1,2} = \dfrac{-3 \pm \sqrt{9+24}}{4} = \underline{\dfrac{-3 \pm \sqrt{33}}{4}}$

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③ $\arccos(\underbrace{-x^2 + 4x - 1}_{g(x)}) > \frac{\pi}{3}$

I $g(x) \in \langle -1, 1 \rangle$. Pokud je splněno I, tak II $\arccos(g(x)) > \arccos\left(\frac{1}{2}\right)$ $\Updownarrow$ arccos je $\Downarrow$

$g(x) < \frac{1}{2}$

Tedy dohromady řešíme: $-1 \overset{(A)}{\leq} g(x) \overset{(B)}{<} \frac{1}{2}$

(A) $x^2 - 4x \leq 0;\ x_{1,2} = \begin{cases} 0 \\ 4 \end{cases} \Rightarrow \underline{x \in \langle 0, 4 \rangle}$

(B) $x^2 - 4x + \frac{3}{2} > 0;\ x_{1,2} = \dfrac{4 \pm \sqrt{16-6}}{2} = 2 \pm \dfrac{\sqrt{10}}{2} \Rightarrow \underline{x \in \left(-\infty, 2 - \frac{\sqrt{10}}{2}\right) \cup \left(2 + \frac{\sqrt{10}}{2}, +\infty\right)}$

$\sqrt{10} \in (3, 4) \Rightarrow \frac{\sqrt{10}}{2} \in (1, 2) \Rightarrow 0 < 2 - \frac{\sqrt{10}}{2} < 2 + \frac{\sqrt{10}}{2} < 4$

Výsledek: $\underline{x \in \left\langle 0, 2 - \frac{\sqrt{10}}{2}\right) \cup \left(2 + \frac{\sqrt{10}}{2}, 4 \right\rangle}$

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④ $a \in \mathbb{R},\ \cos^2 x + 2\sin x < a + 2 \Leftrightarrow \sin^2 x - 2\sin x + a + 1 > 0;\ \sin x = t$

$t^2 - 2t + a + 1 > 0;\ t_{1,2} = \dfrac{2 \pm \sqrt{4 - 4a - 4}}{2} = 1 \pm \sqrt{-a}.$

I $a > 0$: $\underline{\smile}$ $\quad t \in \mathbb{R} \Rightarrow \sin x \in \mathbb{R} \Rightarrow \underline{x \in \mathbb{R}}$

II $a \leq 0$ $\quad \left.\begin{array}{l} t \in (-\infty, 1 - \sqrt{-a}) \cup (1 + \sqrt{-a}, +\infty) \\ t = \sin x \Rightarrow t \in \langle -1, 1 \rangle,\ 1 + \sqrt{-a} \geq 1 \end{array}\right\} \Rightarrow t \in (-\infty, 1 - \sqrt{-a}) \cap \langle -1, 1 \rangle$

$1 - \sqrt{-a} > -1 \Leftrightarrow 2 > \sqrt{-a} \Leftrightarrow 4 > -a \Leftrightarrow a > -4$

i) $a \leq -4$: $t \in \emptyset \Rightarrow \underline{x \in \emptyset}$

ii) $a \in (-4, 0\rangle$: $t \in \langle -1, 1 - \sqrt{-a})$, tedy chceme: $\sin x < 1 - \sqrt{-a}$

- $x \in \langle -\frac{\pi}{2}, \frac{\pi}{2} \rangle$: $\sin x = 1-\sqrt{-a} \iff x = \arcsin(1-\sqrt{-a})$
- $x \in \langle -\frac{3\pi}{2}, -\frac{\pi}{2} \rangle$: — " — $\qquad x = -\pi - \arcsin(1-\sqrt{-a})$

$x \in \langle -\frac{3\pi}{2}, \frac{\pi}{2} \rangle$: $\sin x < 1-\sqrt{-a} \iff x \in (-\pi - \arcsin(1-\sqrt{-a}), \arcsin(1-\sqrt{-a}))$

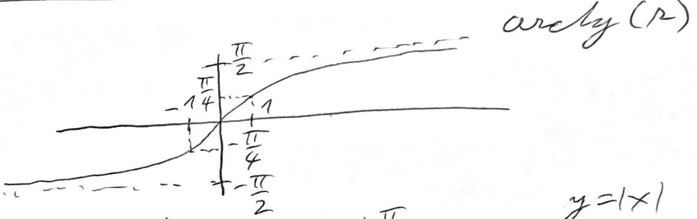
$x \in \mathbb{R}$: $\sin x < 1-\sqrt{-a} \iff x \in \bigcup_{k\in\mathbb{Z}} (-\pi - \arcsin(1-\sqrt{-a}) + 2k\pi, \arcsin(1-\sqrt{-a}) + 2k\pi) = M$

Řešení: $\mathbb{R}$; $a > 0$

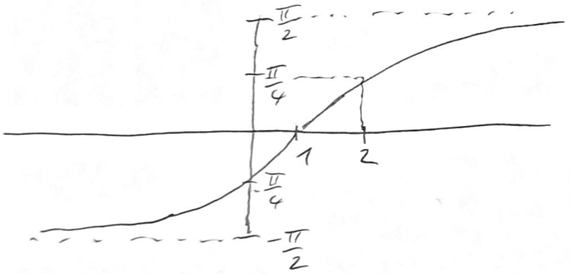
$\emptyset$; $a \leq -4$

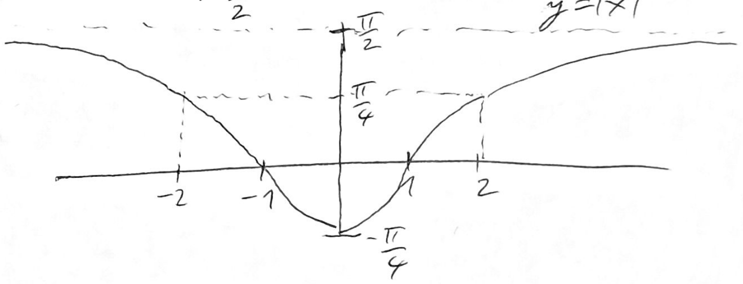
M; $a \in (-4, 0\rangle$

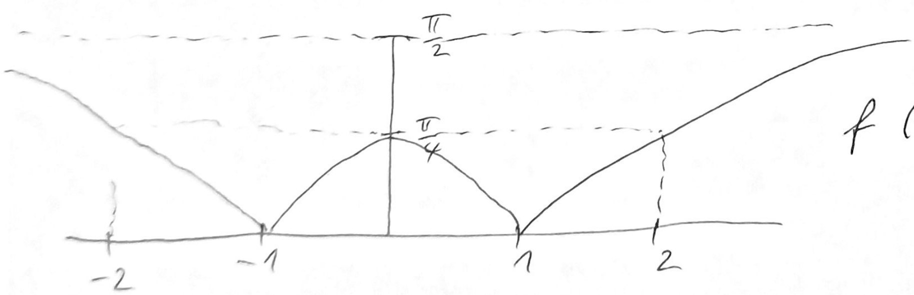
5 $f(x) = |\operatorname{arctg}(\underbrace{|x|-1}_{r})|$



$r = y-1 \qquad \operatorname{arctg}(y-1)$







$f(x)$