

10. cvičení - výsledky

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Příklad 1

- (a) $\mathcal{D}_f = [0, \infty)$; $f'(x) = \frac{\sqrt{3}}{2\sqrt{x}} - 51x^2 + 5x^4$; $\mathcal{D}_{f'} = (0, \infty)$
- (b) $\mathcal{D}_f = \mathbb{R}$; $f'(x) = e^x(1 + x^2)$; $\mathcal{D}_{f'} = \mathbb{R}$
- (c) $\mathcal{D}_f = [0, \infty)$; $f'(x) = 1 + \frac{1}{4x^{3/4}} + \frac{1}{\sqrt{x}}$; $\mathcal{D}_{f'} = (0, \infty)$
- (d) $\mathcal{D}_f = (0, \infty)$; $f'(x) = 1 + 2e^x + \frac{1}{\log 3} + (1 + \frac{1}{\log 3}) \log x$; $\mathcal{D}_{f'} = (0, \infty)$
- (e) $\mathcal{D}_f = \mathbb{R}$; $f'(x) = \log(\frac{3}{2})(\frac{3}{2})^x$; $\mathcal{D}_{f'} = \mathbb{R}$
- (f) $\mathcal{D}_f = \mathbb{R} \setminus \{0\}$; $f'(x) = \frac{9-4x-x^2}{x^4}$; $\mathcal{D}_{f'} = \mathbb{R} \setminus \{0\}$
- (g) $\mathcal{D}_f = \mathbb{R} \setminus \{-6, 1\}$; $f'(x) = \frac{-23-26x+7x^2}{(x-1)^2(6+x)^2}$; $\mathcal{D}_{f'} = \mathbb{R} \setminus \{-6, 1\}$
- (h) $\mathcal{D}_f = (0, \infty)$; $f'(x) = \frac{1}{x} - \frac{\sin x}{\pi}$; $\mathcal{D}_{f'} = (0, \infty)$
- (i) $\mathcal{D}_f = (0, \infty) \setminus \{1\}$; $f'(x) = \frac{2+x \cos(x)(-1+\log(x))-x^2 \log(x) \sin(x)}{x \log^2(x)}$; $\mathcal{D}_{f'} = (0, \infty) \setminus \{1\}$
- (j) $\mathcal{D}_f = \mathbb{R} \setminus (\pi/2 + \pi \mathbb{Z})$; $f'(x) = \frac{1}{\cos^2(x)} - \cos(x)$; $\mathcal{D}_{f'} = \mathcal{D}_f$
- (k) $\mathcal{D}_f = \mathbb{R} \setminus (\pi/2 + \pi \mathbb{Z})$; $f'(x) = \cos^2(x) - \sin^2(x) - \frac{1}{\cos^2(x)}$; $\mathcal{D}_{f'} = \mathcal{D}_f$
- (l) $\mathcal{D}_f = [-1, 1]$; $f'(x) = \frac{1}{\sqrt{1-x^2}} - \frac{3}{(1+x^2)}$; $\mathcal{D}_{f'} = (-1, 1)$
- (m) $\mathcal{D}_f = [-1, 1]$; $f'(x) = -\frac{5}{\sqrt{1-x^2}} + \frac{2}{1+x^2}$; $\mathcal{D}_{f'} = (-1, 1)$

Příklad 2

- (a) $\mathcal{D}_f = \mathbb{R}$; $f'(x) = 78(x^2 + 50x + 12)^{77} \cdot (x^2 + 50x + 12)$; $\mathcal{D}_{f'} = \mathbb{R}$
- (b) $\mathcal{D}_f = \mathbb{R} \setminus \{7\}$; $f'(x) = -\frac{3x^2(x+2)^7(31x+14)}{(x-7)^{-12}}$; $\mathcal{D}_{f'} = \mathbb{R} \setminus \{7\}$
- (c) $\mathcal{D}_f = \mathbb{R}$; $f'(x) = -\frac{1}{1+4x^2} \cdot 2$; $\mathcal{D}_{f'} = \mathbb{R}$
- (d) $\mathcal{D}_f = \mathbb{R}$; $f'(x) = \frac{3+4x-3x^2}{(1+x^2)^2}$; $\mathcal{D}_{f'} = \mathbb{R}$
- (e) $\mathcal{D}_f = \mathbb{R}$; $f'(x) = \frac{2x+1}{x^2+x+2}$; $\mathcal{D}_{f'} = \mathbb{R}$
- (f) $\mathcal{D}_f = \mathbb{R}$; $f'(x) = -\sin(x^3 - x + 2)^9 \cdot 9(x^3 - x + 2)^8 \cdot (3x^2 - 1)$; $\mathcal{D}_{f'} = \mathbb{R}$
- (g) $\mathcal{D}_f = [-2, 2]$; $f'(x) = -\frac{x}{\sqrt{4-x^2}}$; $\mathcal{D}_{f'} = (-2, 2)$
- (h) $\mathcal{D}_f = (-3, 3)$; $f'(x) = 9 \cdot (9 - x^2)^{-\frac{3}{2}}$; $\mathcal{D}_{f'} = (-3, 3)$
- (i) $\mathcal{D}_f = \mathbb{R} \setminus \{-1\}$; $f'(x) = \frac{x(1-x)^2(2-4x^2-4x)}{(1+x)^2}$; $\mathcal{D}_{f'} = \mathbb{R} \setminus \{-1\}$

- (j) $\mathcal{D}_f = \mathbb{R} \setminus \{0\}$; $f'(x) = 2e^{-\frac{1}{x^2}} \frac{1}{x^3}$; $\mathcal{D}_{f'} = \mathbb{R} \setminus \{0\}$
- (k) $\mathcal{D}_f = (0, \infty)$; $f'(x) = x^x \cdot (\log x + 1)$; $\mathcal{D}_{f'} = (0, \infty)$
- (l) $\mathcal{D}_f = (0, \infty)$; $f'(x) = \left(\frac{1}{x}\right)^{\frac{1}{x}} \cdot \left(\frac{-\log x}{x^2} + 1\right)$; $\mathcal{D}_{f'} = (0, \infty)$
- (m) $\mathcal{D}_f = \bigcup_{k \in \mathbb{Z}} (2k\pi, \pi + 2k\pi)$; $f'(x) = e^{\cos x \cdot \log \sin x} \cdot \left(-\sin x \log \sin x + \frac{\cos^2 x}{\sin x}\right)$; $\mathcal{D}_{f'} = \bigcup_{k \in \mathbb{Z}} (2k\pi, \pi + 2k\pi)$
- (n) $\mathcal{D}_f = \mathbb{R}$; $f'(x) = \frac{-\sin x}{\sqrt{1-\cos^2 x}}$; $\mathcal{D}_{f'} = \mathbb{R}$
- (o) $\mathcal{D}_f = [-1, 1)$; $f'(x) = \frac{1}{\arccos x} \cdot \frac{-1}{\sqrt{1-x^2}}$; $\mathcal{D}_{f'} = (-1, 1)$
- (p) $\mathcal{D}_f = \mathbb{R} \setminus \{0\}$; $f'(x) = \frac{-1}{2+x^2}$; $\mathcal{D}_{f'} = \mathbb{R} \setminus \{0\}$
- (q) $\mathcal{D}_f = \mathbb{R}$; $f'(x) = e^{x^2-x+1} \cdot (2x+1) - \sqrt{\frac{e^{2x}+1}{e^{2x}}} \cdot \frac{e^{2x}(1-e^{2x})}{(e^{2x}+1)^2}$; $\mathcal{D}_{f'} = \mathbb{R}$
- (r) $\mathcal{D}_f = \mathbb{R} \setminus (0, 1)$; $f'(x) = \frac{1}{x\sqrt{x^2-1}} - \frac{\sqrt{x^2-1} \cdot \log x \cdot 2x^2}{x^3-x}$; $\mathcal{D}_{f'} = \mathbb{R} \setminus (0, 1)$
- (s) $\mathcal{D}_f = [-1, 1]$; $f'(x) = (\arcsin(x^3))^2 + 2x \cdot \arcsin(x^3) \cdot \frac{3x^2}{\sqrt{1-x^6}}$; $\mathcal{D}_{f'} = (-1, 1)$

Příklad 3

- (a) $\mathcal{D}_f = \mathbb{R}$; $f'(x) = f'(x) = \begin{cases} -1, & x \in (-\infty, 0) \\ 1, & x \in (0, \infty) \end{cases}$; $\mathcal{D}_{f'} = \mathbb{R} \setminus \{0\}$; v $x = 0$ je derivace zleva -1 , zprava 1
- (b) $\mathcal{D}_f = \mathbb{R}$; $f'(x) = f' = \begin{cases} 3-2x, & x \in (-\infty, 0) \cup (3/2, \infty), \\ 2x, & x \in (0, 3/2) \end{cases}$,
 $f'_-(0) = 3, f'_+(0) = 0, f'_-(3/2) = 3, f'_+(3/2) = 0$; $\mathcal{D}_{f'} = \mathbb{R} \setminus \{0, \frac{3}{2}\}$
- (c) $\mathcal{D}_f = \mathbb{R} \setminus \{0\}$; $f'(x) = f' = \begin{cases} 1/x, & x \in (-\infty, -1) \cup (1, \infty), \\ -1/x, & x \in (-1, 0) \cup (0, 1), \end{cases}$,
 $f'_-(-1) = -1, f'_+(-1) = 1, f'_-(0) = \infty, f'_+(0) = -\infty, f'_-(1) = -1, f'_+(1) = 1$; $\mathcal{D}_{f'} = \mathbb{R} \setminus \{-1, 0, 1\}$
- (d) $f'(x) = \operatorname{sgn} x$ na $\mathbb{R} \setminus \{0\}$
- (e) $f' = \begin{cases} 1, & x \in (-\infty, 0], \\ 1/(1+x), & x \in (0, \infty) \end{cases}$,
Pozor. Bod $x = 0$ je třeba vyšetřit zvlášť.

Příklad 4

- (a) $\mathcal{D}_f = (0, \infty) \setminus \{1\}$; $f'(x) = -\frac{1}{x \log^2 x}$; $\mathcal{D}_{f'} = (0, \infty) \setminus \{1\}$
- (b) $\mathcal{D}_f = \mathbb{R}$; $f'(x) = 9 \cos^8(x^3 - x + 2) \cdot (-\sin(x^3 - x + 2)) \cdot (3x^2 - 1)$; $\mathcal{D}_{f'} = \mathbb{R}$

(c) $\mathcal{D}_f = \mathbb{R}; f'(x) = \cos(\sin(\sin(x))) \cdot \cos(\sin(x)) \cdot \cos x; \mathcal{D}_{f'} = \mathbb{R}$

(d) $\mathcal{D}_f = (-\infty, 1) \cup (1, \infty); f'(x) = \frac{8x}{x^4-1}; \mathcal{D}_{f'} = (-\infty, 1) \cup (1, \infty)$

(e) $\mathcal{D}_f = \mathbb{R}; f'(x) = \sin(2x) - 2x \cos(x^2); \mathcal{D}_{f'} = \mathbb{R}$

(f) $\mathcal{D}_f = (1, \infty) \setminus \{e\}; f'(x) = \frac{6}{x \log(x) \log(\log^3(x))}; \mathcal{D}_{f'} = (1, \infty) \setminus \{e\}$

(g) $f' = \begin{cases} 2, & x \in (-\infty, -3) \cup (1, \infty), \\ -2x, & x \in (-3, 1) \\ \text{nedef}, & x \in \{-3, 1\} \end{cases},$
 $f'_-(-3) = 2, f'_+(-3) = 6, f'_-(1) = -2, f'_+(3/2) = 2$

(h) $f' = \begin{cases} -1, & x \in (-\infty, 1], \\ 2x - 3, & x \in (1, 2), \\ 1, & x \in [2, \infty) \end{cases},$

Pozor. Body $x = 1$ a $x = 2$ je třeba vyšetřit zvlášť.

(i) $f' = \begin{cases} -\frac{1}{3}x^{2/3} \cos \frac{1}{\sqrt[3]{x}} + 2x \sin \frac{1}{\sqrt[3]{x}}, & x \neq 0, \\ 0, & x = 0 \end{cases},$

Pozor. Bod $x = 0$ je třeba vyšetřit zvlášť.

(j) $f' = \begin{cases} 0, & x \neq 0, \\ -\infty, & x = 0 \end{cases}$