

## Sample test

Summer semester 2011-12

---

1. Find the solution of the system of linear equations and compute determinant of the matrix of the system.

$$\begin{aligned}x + 2y + 3z + 4t &= 1 \\2x - 2y + 3z - 3t &= -5 \\x + y + z + t &= 5 \\4x + 3y - 5z + 2t &= 3\end{aligned}$$

(10 points)

2. Show that

$$\sin(xy) + \cos(xy) = 1$$

determines at some neighborhood of the point  $[\pi, 0]$  implicitly given function with variable  $x$ . Compute the first and the second derivative of this function at the point  $\pi$ . (10 points)

3. Find extrema of the function  $f$  (if they exist) on the set  $M$ .

$$f(x, y) = x^4y, \quad M = \{[x, y] \in \mathbb{R}^2; x^4 + y^4 \leq 16, x \geq -1\}$$

(15 points)

4. Decide whether the following series is convergent.

$$\sum_{n=1}^{+\infty} \left( \sqrt{n^3 + 1} - \sqrt{n^3 - 1} \right) \quad (10 \text{ points})$$

5. Find primitive function

$$\int \frac{dx}{x^2(x^2 + 1)} \quad (15 \text{ points})$$