

## Matrix calculus

1. Solve the system

$$\begin{aligned}x + 2y - z &= 1 \\2x + 3y &= 1 \\-y + z &= 1\end{aligned}$$

using Gauss elimination and then also using inverse matrix method.

2. Solve the system

$$\begin{aligned}x_1 + 2x_2 - 3x_3 + x_4 &= -5 \\2x_1 + 3x_2 - x_3 + 2x_4 &= 0 \\7x_1 - x_2 + 4x_3 - 3x_4 &= 15 \\x_1 + x_2 - 2x_3 - x_4 &= -3\end{aligned}$$

and compute determinant of the matrix of the system.

3. Compute the rank of the matrix.

$$\begin{pmatrix} 1 & 1 & 1 & 1 & 1 \\ a & 2 & 1 & 2 & a \\ 5 & 6 & 7 & 1 & 3 \\ 1 & 2 & a & 2 & 1 \\ 1 & 0 & 1 & 0 & 1 \end{pmatrix}.$$

4. Compute the determinants.

$$A = \begin{vmatrix} 1 & 2 & 1 & 3 \\ 2 & 4 & 5 & 1 \\ 3 & 6 & 7 & 5 \\ 4 & 8 & 3 & 7 \end{vmatrix}, \quad B = \begin{vmatrix} 246 & 427 & 327 \\ 1014 & 543 & 443 \\ -342 & 721 & 621 \end{vmatrix}.$$

5. Solve the system of linear equations.

$$\begin{aligned}x + y + 2u + 3v &= 1 \\3x + 4y + u + v &= 1 \\2x + 2u + 4v &= 1 \\2x + 5y + u &= 1.\end{aligned}$$

6. Find inverse matrices.

$$\begin{pmatrix} 1 & 2 \\ 2 & 5 \end{pmatrix} \quad \begin{pmatrix} 1 & 2 & 1 \\ 2 & 4 & 2 \\ 1 & 1 & 2 \end{pmatrix} \quad \begin{pmatrix} 1 & 2 & 1 & 0 \\ 2 & 5 & 2 & 1 \\ 1 & 1 & 2 & -1 \\ 0 & 2 & -2 & 3 \end{pmatrix} \quad \begin{pmatrix} 0 & 1 & 1 & 0 & 1 \\ 2 & -1 & 2 & 1 & 3 \\ 1 & 1 & 2 & -1 & 1 \\ 0 & 2 & -2 & 3 & 0 \\ 1 & -1 & 1 & 1 & 1 \end{pmatrix}$$

7. Compute the rank and determinants.

$$\begin{pmatrix} 1 & -1 & 1 & -3 \\ 2 & 1 & a & -1 \\ 1 & 0 & a & -2 \\ 1 & 1 & 2 & -2 \end{pmatrix} \quad \begin{pmatrix} 1 & 0 & 1 & -3 \\ 0 & 1 & 0 & -1 \\ 1 & 0 & a & -2 \\ 1 & a & 2 & -2 \end{pmatrix}$$

8. Compute the rank.

$$\begin{pmatrix} a & -1 & 1 & -3 & 1 & 2 \\ 0 & 2 & 0 & 4 & 1 & -4 \\ -1 & 0 & a & -2 & 1 & 2 \\ a & 1 & 1 & 1 & 2 & -2 \end{pmatrix}$$

9. Solve the following systems of linear equations with parameters.

$$\begin{aligned}x_1 - x_2 + x_3 - 3x_4 &= 1 \\2x_1 + x_2 - x_4 &= -1 \\x_1 - 2x_4 &= -2 \\x_1 + x_2 + 2x_3 - 2x_4 &= -1\end{aligned}$$

$$\begin{aligned}x_1 - x_2 + x_3 - 3x_4 &= 2 \\2x_1 + x_2 - x_4 &= -3 \\x_1 - 2x_4 &= 2 \\x_1 + x_2 + 2x_3 - 2x_4 &= 2\end{aligned}$$

$$\begin{aligned}x_1 - x_2 + x_3 - 3x_4 &= 0 \\2x_1 + x_2 - x_4 &= -1 \\x_1 - 2x_4 &= 0 \\x_1 + x_2 + ax_3 - 2x_4 &= -1\end{aligned}$$

$$\begin{aligned}2x_1 + 2x_2 + x_3 + 3x_4 &= 1 \\-2x_1 + x_2 - 2x_3 + x_4 &= -1 \\ax_1 + ax_2 - 2x_4 &= 0 \\x_1 + x_2 - 2x_4 &= -1\end{aligned}$$

### Solutions

1.  $x = 5, y = -3, z = -2$
2.  $x_1 = 1, x_2 = 0, x_3 = 2, x_4 = 0, \det A = -60$
3.  $h(A) = 4$  for  $a \neq 1$ ;  $h(A) = 3$  for  $a = 1$
4.  $A = 0, B = -29.400.000$
5.  $x = 1/6 - t, y = t, u = 2/3 - 3t, v = -1/6 + 2t, t \in \mathbb{R}$
- 6.1-4.

$$\begin{pmatrix} 5 & -2 \\ -2 & 1 \end{pmatrix} \quad \text{Inverse matrix does not exist.} \quad \begin{pmatrix} 4 & -3 & 3 & 2 \\ 0 & 1 & -2 & -1 \\ -3 & 1 & 1 & 0 \\ -2 & 0 & 2 & 1 \end{pmatrix}$$

$$\begin{pmatrix} -5/4 & 1/4 & 3/4 & 1/4 & -1/4 \\ 1/16 & -1/16 & 5/16 & 3/16 & -3/16 \\ 5/8 & -5/8 & 1/8 & -1/8 & 9/8 \\ 3/8 & -3/8 & -1/8 & 1/8 & 7/8 \\ 5/16 & 11/16 & -7/16 & -1/16 & -15/16 \end{pmatrix}$$

- 7.1.  $h(A) = 4$  for  $a \neq 7/6$ ;  $h(A) = 3$  for  $a = 7/6$ ;  $\det A = -6a + 7$
- 7.2.  $h(A) = 4$  for  $a \neq \pm\sqrt{2}$ ;  $h(A) = 3$  for  $a \in \{\sqrt{2}, -\sqrt{2}\}$ ;  $\det A = a^2 - 2$
8.  $h(A) = 3$
- 9.1.  $x_1 = 2, x_2 = -3, x_3 = 2, x_4 = 2$
- 9.2.  $x_1 = -4, x_2 = 2, x_3 = -1, x_4 = -3$
- 9.3. If  $a \neq -3/2$ , then  $x_1 = -2a/(3+2a), x_2 = (a-3)/(3+2a), x_3 = -3/(3+2a), x_4 = -a/(3+2a)$ ;  
if  $a = -3/2$ , then there is no solution.
- 9.4. If  $a \neq 1$ , then  $x_1 = \frac{5a+12}{6(a-1)}, x_2 = -\frac{5a+6}{6(a-1)}, x_3 = -\frac{a+6}{2(a-1)}, x_4 = \frac{a}{2(a-1)}$ ; if  $a = 1$ , then the system has no solution.